

Current Topics in Statistics for Applied Researchers

Factor Analysis

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Purpose

- to describe and demonstrate factor analysis of survey instrument data
 - primarily for assessment of established scales
 - with some discussion of the development of new scales
- emphasizing its use in exploratory, data-driven analyses
 - called exploratory factor analysis (EFA)
- but with examples of its use in confirmatory, theory-driven analyses
 - called confirmatory factor analysis (CFA)
- using the Statistical Package for the Social Sciences (SPSS) and the Statistical Analysis System (SAS)
 - PDF copy of slides are available on the Internet at

<http://www.ohsu.edu/son/faculty/knafl/factoranalysis.html>

Overview

1. examples of established scales
2. principal component analysis vs. factor analysis
terminology and some primary factor analysis methods
3. factor extraction
survey of alternative methods
4. factor rotation
interpreting the results in terms of scales
5. factor analysis model evaluation
evaluating alternatives for factor extraction and rotation
6. a case study in ongoing scale development
with assistance from Kathleen Knafl

including example analyses in SPSS and SAS

Part 1

Examples of Scales

Data Used in Factor Analysis

- factor analysis is used to identify dimensions underlying response (outcome) variables y
 - observed values for the variables y are available, so they are called manifest variables
 - standardized variables z for the y are typically used
 - and the correlation matrix $\mathbf{R}$ for the z is modeled
- dimensions correspond to variables F called factors
 - observed values for the variables F are not available and so they are called latent variables
- most types of manifest variables can be used
 - but more appropriate if they have more than a few distinct values and an approximate bell-shaped distribution
- factor analysis is used in many different application areas
 - in the health sciences, it is usually applied to survey instrument data, and so that is the focus of these notes

A Simple Example

- subjects undergoing radiotherapy were measured on 6 dimensions [1, p. 33]
 - number of symptoms
 - amount of activity
 - amount of sleep
 - amount of food consumed
 - appetite
 - skin reaction
- can these be grouped into sets of related measures to obtain a more parsimonious description of what they represent?
 - perhaps there are really only 2 distinct dimensions for these 6 variables?

Survey Instruments

- survey instruments consist of items
 - with discrete ranges of values, e.g., 1, 2, ...
- items are grouped into disjoint sets
 - corresponding to scales
 - items in these sets might be just summed
 - and then the scales are called summated
 - possibly after reverse coding values for some items
 - or weighted and then summed
- items might be further grouped into subsets
 - corresponding to subscales
 - the subscales are often just used as the first step in computing the scales rather than as separate measures

Example 1 - SDS

- symptom distress scale [2]
 - symptom assessment for adults with cancer
 - 13 items scored 1,2,3,4,5 measuring distress experience related to severity of 11 symptoms
 - nausea, appetite, insomnia, pain, fatigue, bowel pattern, concentration, appearance, outlook, breathing, cough
 - and the frequency as well for nausea and pain
 - 1 total scale
 - sum of the 13 items with none reverse coded
 - higher scores indicate higher levels of symptom distress

Example 2 - CDI

- Children's Depression Inventory [3]
 - 27 items scored 0,1,2 assessing aspects of depressive symptoms for children and adolescents
 - 1 total scale
 - sum of the 27 items after reverse coding 13 of them
 - higher scores indicate higher depressive symptom levels
 - 5 subscales measuring different aspects of depressive symptoms
 - negative mood, interpretation problems, ineffectiveness, anhedonia, and negative self-esteem
 - the total scale equals the sum of the subscales
 - total scale used in practice rather than subscales

Example 3 – FACES II

- Family Adaptability & Cohesion Scales [4]
 - has several versions, will consider version II
 - 30 items scored 1,2,3,4,5
 - 2 scales
 - family adaptability
 - family's ability to alter its role relationships and power structure
 - sum of 14 of the items after reverse coding 2 of them
 - higher scores indicate higher family adaptability
 - family cohesion
 - the emotional bonding within the family
 - sum of the other 16 of the items after reverse coding 6 of them
 - higher scores indicate higher family cohesion
 - 2 scales are typically used separately, but are sometimes summed to obtain a total FACES scale

Example 4 - DQOLY

- Diabetes Quality of Life – Youth scale [5]
 - 51 items scored 1,2,3,4,5
 - 3 scales
 - impact of diabetes
 - sum of 23 of the items after reverse coding 1 of them
 - higher scores indicate higher negative impact (worse QOL)
 - diabetes-related worries
 - sum of 11 other items with none reverse coded
 - higher scores indicate more worries (worse QOL)
 - satisfaction with life
 - sum of the other 17 items with none reverse coded
 - higher scores indicate higher satisfaction (better QOL)
 - so it has the reverse orientation to the other scales
 - the 3 scales are typically used separately and not usually combined into a total scale
- the youth version of the scale is appropriate for children 13-17 years old
 - also has a school age version for children 8-12 years old and a parent version

Example 5 - FACT

- Functional Assessment of Cancer Therapy [6]
 - 27 general (G) items scored 0-4
 - 4 subscales
 - physical, social/family, emotional, functional subscales
 - sums of 6-7 of the general items with some reverse coded
 - 1 scale
 - the functional well-being scale (FACT-G)
 - the sum of the 4 subscales
 - higher scores indicate better levels of quality of life
- extra items available for certain types of cancers
 - 7 for colon (C) cancer, 9 for lung (L) cancer, scored 0-4
 - summed with some reverse coded into separate scales (FACT-C/FACT-L)
 - these can also be added to the FACT-G
 - an overall functional well-being measure specific to the type of cancer
- has been extended to chronic illnesses (FACIT)

Example 6 – MOS SF-36

- Medical Outcomes Study Short Form – 36 [7]
 - 36 items scored in varying ranges
 - 8 subscales computed from 35 of the items
 - physical functioning, role-physical, bodily pain, general health, vitality, role-emotional, social functioning, mental health
 - 2 scales computed from different weightings of the 8 subscales
 - two dimensions of quality of life
 - physical component scale (PCS) – physical health
 - mental component scale (MCS) – mental health
 - 1 other item reporting overall assessment of health
 - but not used in computing scales
- other versions with 12, 20, and 116 items

Example 7 - FMSS

- Family Management Style Survey
 - a survey instrument currently under development
 - parents of children having a chronic illness are being interviewed on how their families manage their child's chronic illness
 - as many parents as are willing to participate
- there are 65 initial FMSS items
 - items 1-57 are applicable to both single and partnered parents
 - items 58-65 address issues related to the parent's spouse and so are not completed by single parents
- all items are coded from 1-5
 - 1="strongly disagree" and 5="strongly agree"
- challenge is to account for inter-parental correlation¹⁴

Scale Development/Assessment

- as part of scale development, an initial set of items is reduced to a final set of items which are then combined into one or more scales and possibly also subscales
- established scales, when used in novel settings, need to be assessed for their applicability to those settings
- such issues can be addressed in part using factor analysis techniques
 - will address these using data for the CDI, FACES II, DQOLY, and FMSS instruments
 - starting with a popular approach related to principal component analysis (PCA)

Part 2

Principal Component Analysis vs. Factor Analysis

- factors, factor scores, and loadings
- eigenvalues and total variance
- conventions for choosing the # of factors
- communalities and specificities
- example analyses

Principal Component Analysis

- standardize each item y
 - $z = (y - \text{its average}) / (\text{its standard deviation})$
 - so the variance of each z equals 1
 - and the sum of the variances for all z 's equals the # of items
 - called the total variance
 - items are typically standardized, but they do not have to be
- associated with the z 's are an equal # of principal components (PC's)
- each PC can be expressed as a weighted sum of z 's
 - this is how they are defined and used for a standard PCA
- each z can be expressed as a weighted sum of PC's
 - this is how they are used in a factor analysis based on PC's

Variable Reduction

- PCA can be used to reduce the # of variables
- one such use is to simplify a regression analysis by reducing the # of predictor variables
 - predict a dependent variable using the first few PC's determined from the predictors, not all predictors
- similar simplification for factor analysis
 - use the first few factors to model the z's
- but not clear how many should you use
 - i.e., how many factors to extract?
- diminishing returns to using more factors (or PC's), but hopefully there is a natural separation point

Radiotherapy Data

- can we model the correlation matrix **R** as if its 6 dimensions were determined by 2 factors?
 - skin reaction is related to none of the others while appetite is related to the other 4 variables

Correlations

		Number of Symptoms	Amount of Activity	Amount of Sleep	Amount of Food Consumed	Appetite	Skin Reaction
Number of Symptoms	Pearson Correlation	1	.842**	.322	.412	.766**	.348
	Sig. (2-tailed)		.002	.364	.237	.010	.325
	N	10	10	10	10	10	10
Amount of Activity	Pearson Correlation	.842**	1	.451	.610	.843**	-.116
	Sig. (2-tailed)	.002		.191	.061	.002	.749
	N	10	10	10	10	10	10
Amount of Sleep	Pearson Correlation	.322	.451	1	.466	.641*	.005
	Sig. (2-tailed)	.364	.191		.174	.046	.989
	N	10	10	10	10	10	10
Amount of Food Consumed	Pearson Correlation	.412	.610	.466	1	.811**	.067
	Sig. (2-tailed)	.237	.061	.174		.004	.854
	N	10	10	10	10	10	10
Appetite	Pearson Correlation	.766**	.843**	.641*	.811**	1	.102
	Sig. (2-tailed)	.010	.002	.046	.004		.778
	N	10	10	10	10	10	10
Skin Reaction	Pearson Correlation	.348	-.116	.005	.067	.102	1
	Sig. (2-tailed)	.325	.749	.989	.854	.778	
	N	10	10	10	10	10	10

** . Correlation is significant at the 0.01 level (2-tailed).

* . Correlation is significant at the 0.05 level (2-tailed).

(Common) Factor Analysis

- treat each z as equal to a weighted sum of the same k factors F plus an error term u that is unique to each z
 - the weights L are called loadings
$$z = L(1) \cdot F(1) + L(2) \cdot F(2) + \dots + L(k) \cdot F(k) + u$$
- the factors F are unobservable, so need to estimate their values
 - called the factor scores FS
- same approach used with any factor extraction method
- since the same k factors F are used with each z , they are called common factors
- but different loadings L are used with each z
- different or unique errors u are also used with each z
 - hence they are called the unique (or specific) factors

Factor Analysis Assumptions

- the factor analysis model for the standardized items z satisfies

$$z = L(1) \cdot F(1) + L(2) \cdot F(2) + \dots + L(k) \cdot F(k) + u$$

- assuming also that
 - the common factors F are
 - standardized (with mean 0 and variance 1) and
 - independent of each other
 - the unique (specific) factors u
 - have mean zero (but not necessarily variance 1) and
 - are independent of each other
 - all common factors are independent of all unique factors

Factor Analysis Using PC's

- PCA produces weights for computing the principal components PC from the z's
- factor analysis based on PC's uses these weights and PC scores to produce factor loadings L and factor scores FS to estimate factors, but only the first k are used

$$z = L(1) \cdot FS(1) + L(2) \cdot FS(2) + \dots + L(k) \cdot FS(k) + u$$

- loadings are combined as entries in a matrix called the factor (pattern) matrix
 - 1 row for each standardized item z
 - each containing loadings on all k factors for that standardized item
 - 1 column for each factor F
 - each containing loadings for all z's on that factor

Radiotherapy Data Loadings

- extracted 2 factors using the PCs
- # of symptoms loads more highly (.827) on factor 1 than on factor 2 (.361)
 - but the loading on factor 2 is not that small so maybe # of symptoms is distinctly related to both factors
- loadings are usually rotated and ordered to be better able to allocated them to factors

Component Matrix^a

	Component	
	1	2
Number of Symptoms	.827	.361
Amount of Activity	.903	-.152
Amount of Sleep	.659	-.230
Amount of Food Consumed	.790	-.128
Appetite	.977	-.037
Skin Reaction	.134	.955

Extraction Method: Principal Component Analysis.

a. 2 components extracted.

Ordered Rotated Loadings

- the first 5 variables load more highly on factor 1 than on factor 2
- only skin reaction loads more highly on factor 2 than factor 1
 - but factors with only 1 associated variable are suspect
- however, # of symptoms loads highly on both factors
 - maybe it should be discarded since it is not unidimensional?

Rotated Component Matrix ^a

	Component	
	1	2
Appetite	.968	.140
Amount of Activity	.915	.015
Amount of Food Consumed	.801	.017
Number of Symptoms	.748	.505
Amount of Sleep	.690	-.107
Skin Reaction	-.041	.963

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 3 iterations.

Communalities

- part of each z is explained by the common factors

$$z = L(1) \cdot F(1) + L(2) \cdot F(2) + \dots + L(k) \cdot F(k) + u$$

- the communality for z is the amount of its variance explained by the common factors (hence its name)

$$1 = \text{VAR}[z] = \text{VAR}[L(1) \cdot F(1) + L(2) \cdot F(2) + \dots + L(k) \cdot F(k)] + \text{VAR}[u]$$

– variances add up due to independence assumptions

- the variance of the unique factor u is called the uniqueness

$$1 = \text{VAR}[z] = \text{communality} + \text{uniqueness}$$

so the communality is between 0 and 1

- u is also called the specific factor for z and then its variance is called the specificity

PC-Based Factor Analysis

- can extract any # k of factors F up to the # of items z
- when $k =$ the # of items
 - use all the factors F (and PC's)
 - so the communality=1 and the uniqueness=0 for all z
 - not really a factor analysis
- when $k <$ the # of z 's
 - communalities are determined from loadings for the k factors
 - the communality of $z =$ the sum of the squares of the loadings for z over all the factors F
 - then subtracted from 1 to get the uniqueness for z
 - but need initial values for the communalities to start the computations

The PC Method

- start by setting all communalities equal to 1
 - they stay that way if all the factor scores are used
- if the # of factors $<$ the # of items
 - recompute the communalities based on the extracted factors

Radiotherapy Data Communalities

- communalities started out as all 1's
 - since the PC method was used to extract factors
- but they were re-estimated based on loadings for the 2 extracted factors
 - the new values are < 1 as they should be when the # of factors $<$ the # of items

Communalities

	Initial	Extraction
Number of Symptoms	1.000	.814
Amount of Activity	1.000	.838
Amount of Sleep	1.000	.488
Amount of Food Consumed	1.000	.641
Appetite	1.000	.956
Skin Reaction	1.000	.930

Extraction Method: Principal Component Analysis.

Initial Communalities

- the principal component (PC) method
 - all communalities start out as 1
 - and are then recomputed from the extracted factors
- the principal factor (PF) method
 - the initial communalities are estimated
 - and are then recomputed from the extracted factors
- for both of these, can stop after the first step or iterate the process until the communalities do not change much
 - a problem occurs when communalities come out larger than 1 though

Initial Communalities Estimates

- initial communalities are usually estimated using the squared multiple correlations
 - square the multiple correlation of each z with all the other z 's
- SAS supports alternative ways to estimate the initial communalities
 - but calls them prior communalities
 - adjusted SMCs
 - divide the SMCs by their maximum value
 - maximum absolute correlations
 - use the maximum absolute correlation of each z with all the other z 's
 - random settings
 - generate random numbers between 0 and 1
 - not available in SPSS

PC-Based Alternatives

- 1-step principal component (PC) method
 - set communalities all to an initial value of 1
 - compute loadings and factor scores
 - re-estimate the communalities from these and stop
 - iterated version available in SAS but not in SPSS
- 1-step principal factor (PF) method
 - estimate the initial values for the communalities
 - compute loadings and factor scores
 - re-estimate the communalities from these and stop
 - 1-step procedure available in SAS but not in SPSS
 - iterated version available in both SPSS and SAS
 - called principal axis factoring (PAF) in SPSS

Eigenvalues

- each factor F (or PC or FS) has an associated eigenvalue EV
 - also called a characteristic root since by definition it is a solution to the so-called characteristic equation for the correlation matrix $\mathbf{R}$
- the sum of the eigenvalues over all factors equals the total variance
 - sum of the EV 's = total variance = # of items
 - so an eigenvalue measures how much of the total variance of the z 's is accounted for by its associated factor (or PC)
 - in other words, factors with larger eigenvalues contribute more towards explaining the total variance of the z 's
- eigenvalues are generated in decreasing order
 - $EV(1) \geq EV(2) \geq EV(3) \geq \dots$
 - eigenvalues at the start have the more important factors (or PC's)

The Eigenvalue-One Rule

- the eigenvalue-one (EV-ONE) rule
 - also called the Kaiser-Guttman rule
- says to use the factors with eigenvalues > 1 and discard the rest
- an eigenvalue > 1 means its factor contributes more to the total variance than a single z
 - since each z has variance 1 and so contributes 1 to the total variance

Radiotherapy Data Eigenvalues

- EV-ONE says to extract 2 factors
- 2 factors explain about 78% of the total variance

Total Variance Explained

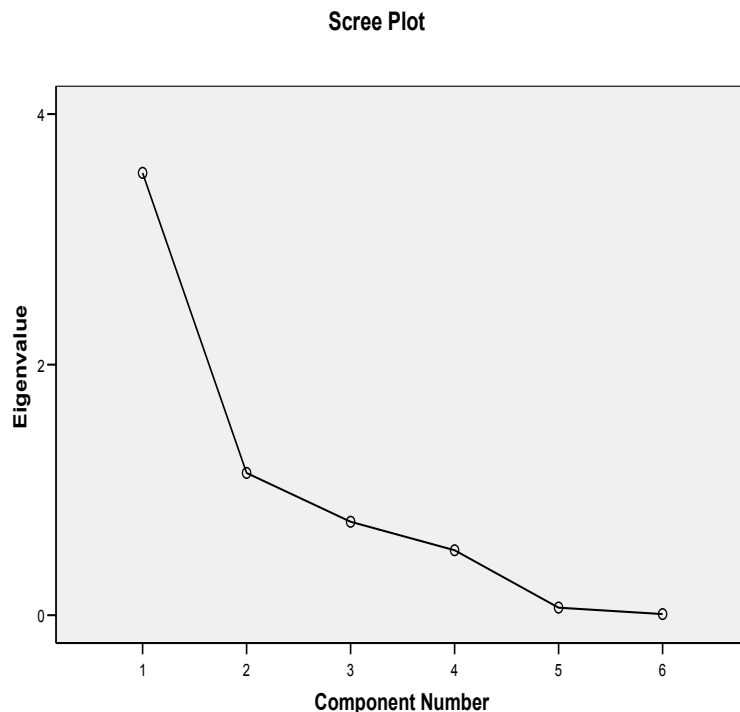
Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.531	58.844	58.844	3.531	58.844	58.844
2	1.136	18.927	77.770	1.136	18.927	77.770
3	.746	12.432	90.202			
4	.519	8.642	98.844			
5	.061	1.010	99.855			
6	.009	.145	100.000			

Extraction Method: Principal Component Analysis.

Other Possible Selection Rules

- individual % of the total variance
 - use the factors whose eigenvalues exceed 5% (or 10%) of the total variance [8]
- cumulative % of the total variance
 - use initial subset of factors the sum of whose eigenvalues first exceeds 70% (or 80%) of the total variance [8]
- inspect a scree plot for a big change in slope
 - the plot of the eigenvalues in decreasing order
- same rules apply to reducing the # of PC's

Radiotherapy Data Scree Plot



- "scree" means debris at the bottom of a cliff
 - look for the point on x-axis separating the "cliff" from the "debris" at its bottom
 - i.e., a large change in slope
- biggest change is between 1 and 2
 - perhaps there is only 1 factor?
 - or maybe as much as 4

Factor Analysis Properties

- the loading L of z on F is the correlation between z and F
- the square of the loading L is the portion of the variance of z explained by F
- the sum of the square loadings over all factors is the portion of the variance of z explained by all the factors
 - so this sum equals the communality of z
- the sum of the squared loadings over all z is the portion of the total variance explained by F
 - so this sum equals the eigenvalue EV for F
- the correlation between any 2 z 's is the sum of the products of their loadings on each of the factors

Factor Analysis Types

- exploratory factor analysis (EFA)
 - use the data to determine how many factors there should be and which items to associate with those factors
 - can be accomplished using the PC method, the PF method, and a variety of other methods
 - supported by SPSS and SAS
 - use Analyze/Data Reduction/Factor... in SPSS
 - use PROC FACTOR in SAS
- confirmatory factor analysis (CFA)
 - use theory to pre-specify an item-factor allocation and assess whether it is a reasonable choice
 - supported by SAS but not by SPSS
 - use PROC CALIS (**C**ovariance **A**na**L**ysis) in SAS
 - SPSS users need to use another tool like LISREL or AMOS

The ABC Survey Instrument Data

- example factor analyses are presented of
 - the baseline CDI, FACES II, and DQOLY items
 - without prior reverse coding
 - for the 103 adolescents with type 1 diabetes who responded at baseline to all the items of all 3 of these instruments
 - 88.0% of the 117 subjects providing some baseline data
 - from Adolescents Benefit from Control (ABCs) of Diabetes Study (Yale School of Nursing, PI Margaret Grey) [9]
- using SPSS (version 14.2) and SAS (version 9.1)
 - data and code are available on the Internet at
<http://www.ohsu.edu/son/faculty/knafl/factoranalysis.html>
- see [10] for details for some of the reported results

Principal Component Example

- in SPSS, run the PC method for the FACES items extracting 2 factors and generate a scree plot
 - the same as the recommended # of scales
 - click on Analyze/Data Reduction/Factor...
 - set "Variables:" to FACES1-FACES30
 - in "Extraction...", set "Number of factors" to 2 and request a scree plot
 - use the default method of "Principal components"
 - then execute the analysis

Communalities

Communalities

	Initial	Extraction
FACES1	1.000	.504
FACES2	1.000	.375
FACES3	1.000	.426
FACES4	1.000	.214
FACES5	1.000	.305
FACES6	1.000	.236
FACES7	1.000	.458
FACES8	1.000	.623
FACES9	1.000	.258
FACES10	1.000	.378
FACES11	1.000	.211
FACES12	1.000	.122
FACES13	1.000	.128
FACES14	1.000	.214
FACES15	1.000	.394
FACES16	1.000	.458
FACES17	1.000	.430
FACES18	1.000	.550
FACES19	1.000	.473
FACES20	1.000	.357
FACES21	1.000	.342
FACES22	1.000	.461
FACES23	1.000	.494
FACES24	1.000	.200
FACES25	1.000	.599
FACES26	1.000	.542
FACES27	1.000	.309
FACES28	1.000	.225
FACES29	1.000	.383
FACES30	1.000	.466

- the initial communalities are all set to 1 for the PC method
- they are then recomputed (in the "Extraction" column) based on the 2 extracted factors
- all the recomputed communalities are < 1 as they should be for a factor analysis with $k < 30$
- if 30 factors had been extracted, the communalities would have all stayed 1
 - a standard PCA

Extraction Method: Principal Component Analysis.

Loadings

Component Matrix^a

	Component	
	1	2
FACES1	.702	.110
FACES2	.611	-.037
FACES3	-.327	.565
FACES4	.454	.093
FACES5	.550	.048
FACES6	.356	.330
FACES7	.677	-.004
FACES8	.789	-.027
FACES9	-.318	.396
FACES10	.338	.513
FACES11	.387	.246
FACES12	-.335	.102
FACES13	.303	.191
FACES14	.185	.424
FACES15	-.231	.584
FACES16	.630	.247
FACES17	.655	-.031
FACES18	.689	.273
FACES19	-.487	.486
FACES20	.565	.195
FACES21	.564	.155
FACES22	.673	.088
FACES23	.686	-.151
FACES24	-.315	.317
FACES25	-.532	.562
FACES26	.731	.089
FACES27	.527	.178
FACES28	-.239	.410
FACES29	-.495	.371
FACES30	.647	.217

Extraction Method: Principal Component Analysis.

a. 2 components extracted.

- the matrix of loadings
 - called the component matrix in SPSS for the PC method
 - 30 rows, 1 for each item z
 - 2 columns, 1 for each factor F
- FACES1 loads much more highly on the first factor than on the second factor
 - since .702 is much larger than .110
 - and so FACES1 is said to be a marker item (or salient) for factor 1

Eigenvalues

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	8.360	27.867	27.867	8.360	27.867	27.867
2	2.777	9.255	37.122	2.777	9.255	37.122
3	1.804	6.012	43.134			
4	1.593	5.309	48.443			
5	1.413	4.712	53.155			
6	1.305	4.350	57.505			
7	1.266	4.221	61.726			
8	1.150	3.835	65.560			
9	.984	3.279	68.839			
10	.898	2.992	71.831			
11	.818	2.726	74.557			
12	.770	2.567	77.124			
13	.708	2.359	79.484			
14	.681	2.268	81.752			
15	.583	1.945	83.697			
16	.563	1.876	85.573			
17	.519	1.731	87.304			
18	.481	1.604	88.908			
19	.453	1.509	90.417			
20	.407	1.357	91.774			
21	.381	1.270	93.043			
22	.361	1.204	94.248			
23	.310	1.035	95.282			
24	.280	.933	96.215			
25	.251	.836	97.051			
26	.226	.752	97.803			
27	.209	.697	98.500			
28	.192	.641	99.141			
29	.155	.516	99.657			
30	.103	.343	100.000			

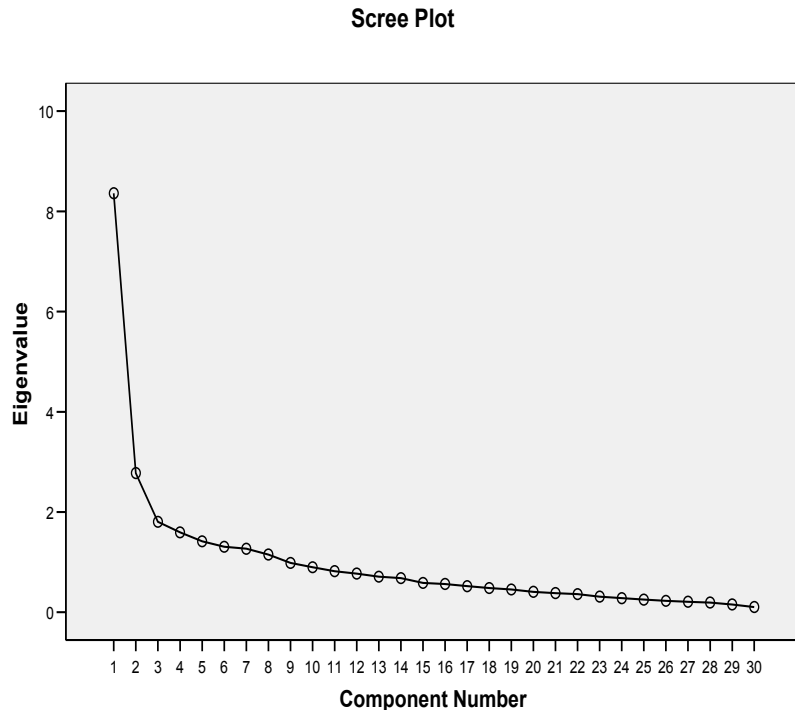
- the "Total" column gives the eigenvalues in decreasing order
- the first 2 factors explain about 28% and 9% individually of the total variance
 - total variance = 30 since items are standardized
- but only 37% together
 - could more be needed?

Extraction Method: Principal Component Analysis.

The # of Factors to Extract

- conventional selection rules give different #'s of factors
 - first 8 have eigenvalues > 1
 - first 4 each explain more than 5% each
 - first 1 each explain more than 10% each
 - first 10 combined explain just over 70%
 - first 14 combined explain just over 80%
- none choose the recommended # of 2 factors

The Scree Plot



- seems to be a large change in slope between 2-3 factors
 - suggests that the recommended # of 2 factors might be a reasonable choice for the ABC FACES items
 - but maybe the slope isn't close to constant until later

Principal Axis Factoring Example

- in SPSS, run the PAF method for the FACES items extracting 2 factors as before
 - re-enter Analyze/Data Reduction/Factor...
 - in "Extraction...", set "Method:" to "Principal axis factoring"
 - note that the default is to analyze the correlation matrix
 - i.e, factor analyze the standardized FACES items z
 - then re-execute the analysis

Communalities

Communalities

	Initial	Extraction
FACES1	.650	.478
FACES2	.538	.343
FACES3	.615	.352
FACES4	.575	.182
FACES5	.594	.272
FACES6	.405	.178
FACES7	.582	.427
FACES8	.702	.609
FACES9	.501	.182
FACES10	.501	.308
FACES11	.511	.161
FACES12	.379	.101
FACES13	.410	.102
FACES14	.427	.129
FACES15	.462	.288
FACES16	.619	.428
FACES17	.699	.399
FACES18	.708	.531
FACES19	.574	.429
FACES20	.504	.321
FACES21	.617	.307
FACES22	.663	.433
FACES23	.723	.462
FACES24	.361	.150
FACES25	.665	.589
FACES26	.586	.522
FACES27	.534	.268
FACES28	.515	.154
FACES29	.489	.331
FACES30	.582	.437

- the initial communalities are all estimated using associated squared multiple correlations
- they are then recomputed based on the 2 extracted factors
- all the initial and recomputed communalities are < 1 as they should be for a factor analysis with $k < 30$

Extraction Method: Principal Axis Factoring.

Loadings

Factor Matrix^a

	Factor	
	1	2
FACES1	.683	.107
FACES2	.585	-.035
FACES3	-.314	.503
FACES4	.423	.055
FACES5	.521	.031
FACES6	.332	.259
FACES7	.653	.001
FACES8	.780	-.025
FACES9	-.299	.304
FACES10	.322	.452
FACES11	.361	.176
FACES12	-.310	.070
FACES13	.281	.152
FACES14	.170	.316
FACES15	-.219	.490
FACES16	.610	.238
FACES17	.631	-.034
FACES18	.676	.272
FACES19	-.471	.455
FACES20	.538	.175
FACES21	.537	.137
FACES22	.651	.096
FACES23	.667	-.133
FACES24	-.294	.253
FACES25	-.525	.560
FACES26	.715	.100
FACES27	.498	.142
FACES28	-.223	.324
FACES29	-.473	.328
FACES30	.627	.208

Extraction Method: Principal Axis Factoring
a. 2 factors extracted. 5 iterations required

- the matrix of loadings
 - 30 rows, 1 for each item z
 - 2 columns, 1 for each factor F
 - SPSS calls it the factor matrix
 - SAS calls it the factor pattern matrix
- FACES1 again loads much more highly on the first factor
 - since .683 is much larger than .107
 - loadings have changed, but only a little
 - from .702 and .110 for the PC method

PC vs. PF Methods

- the use of the PC method vs. the PF method is thought to usually have little impact on the results
 - "one draws almost identical inferences from either approach in *most* analyses" [11, p. 535]
- so far there seems to be only a minor impact to the choice of factor extraction method on the loadings for the FACES data
 - we will continue to consider this issue

Eigenvalues

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	8.360	27.867	27.867	8.360	27.867	27.867
2	2.777	9.255	37.122	2.777	9.255	37.122
3	1.804	6.012	43.134			
4	1.593	5.309	48.443			
5	1.413	4.712	53.155			
6	1.305	4.350	57.505			
7	1.266	4.221	61.726			
8	1.150	3.835	65.560			
9	.984	3.279	68.839			
10	.898	2.992	71.831			
11	.818	2.726	74.557			
12	.770	2.567	77.124			
13	.708	2.359	79.484			
14	.681	2.268	81.752			
15	.583	1.945	83.697			
16	.563	1.876	85.573			
17	.519	1.731	87.304			
18	.481	1.604	88.908			
19	.453	1.509	90.417			
20	.407	1.357	91.774			
21	.381	1.270	93.043			
22	.361	1.204	94.248			
23	.310	1.035	95.282			
24	.280	.933	96.215			
25	.251	.836	97.051			
26	.226	.752	97.803			
27	.209	.697	98.500			
28	.192	.641	99.141			
29	.155	.516	99.657			
30	.103	.343	100.000			

Extraction Method: Principal Component Analysis.

- exactly the same as for the PC method
- in SPSS, eigenvalues are always computed using the PC method
 - even if a different factor extraction method is used
- so always get the same choice for the # of factors with the EV-ONE rule and other related rules
 - but the factor loadings will change

EV-ONE Rule for FACES

- in SPSS, run the PAF method for the FACES items extracting the # of factors determined by the EV-ONE rule
 - re-enter Analyze/Data Reduction/Factor...
 - in "Extraction...", click on "Eigenvalues over:" and leave the default value at 1
 - this was the original default way for choosing # of factors to extract
 - SPSS is set up to encourage the use of the EV-ONE rule
 - then re-execute the analysis

Communalities

Communalities	
	Initial
FACES1	.650
FACES2	.538
FACES3	.615
FACES4	.575
FACES5	.594
FACES6	.405
FACES7	.582
FACES8	.702
FACES9	.501
FACES10	.501
FACES11	.511
FACES12	.379
FACES13	.410
FACES14	.427
FACES15	.462
FACES16	.619
FACES17	.699
FACES18	.708
FACES19	.574
FACES20	.504
FACES21	.617
FACES22	.663
FACES23	.723
FACES24	.361
FACES25	.665
FACES26	.586
FACES27	.534
FACES28	.515
FACES29	.489
FACES30	.582

Extraction Method: Principal Axis Factoring.

- the initial communalities are all estimated using associated squared multiple correlations
 - and so they are the same as before
- but communalities based on the extraction as well as the factor matrix are not produced
- the procedure did not converge because communalities over 1 were generated
 - suggests that the EV-ONE rule is of questionable value for the ABC FACES items

Factor Matrix^a

a. Attempted to extract 8 factors. In iteration 25, the communality of a variable exceeded 1.0. Extraction was terminated.

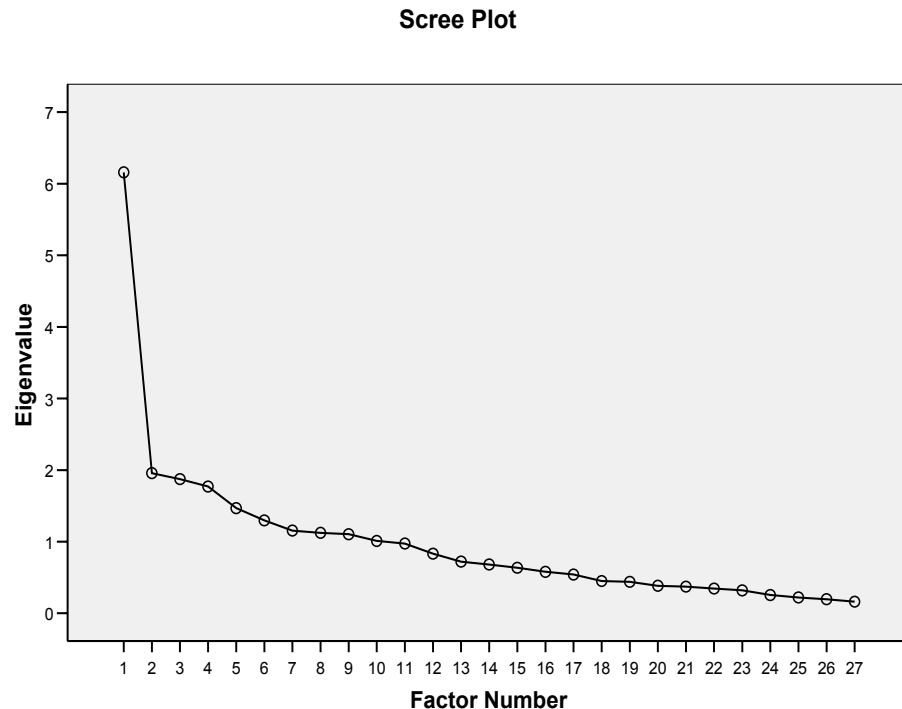
Communalities Anomalies

- communalities are by definition between 0 & 1
- but factor extraction methods can generate communalities > 1
 - Heywood case: when a communality = 1
 - ultra-Heywood case: when a communality > 1
- SAS has an option that changes any communalities > 1 to 1, allowing the iteration process to continue and so avoiding the convergence problems reported for SPSS

EV-ONE Rule for CDI

- in SPSS, run the PAF method for CDI items extracting the # of factors determined by the EV-ONE rule
 - re-enter Analyze/Data Reduction/Factor...
 - from "Variables:", first remove FACES1-FACES30 and then add in CDI1-CDI27
 - then re-execute the analysis
- the EV-ONE rule selects 10 factors
- PAF did not converge in the default # of 25 iterations
 - but the # of iterations can be increased
 - in "Extraction..." change "Maximum Iterations for Convergence:" to 200 (it did not converge at 100)
- after more iterations, extraction is terminated because some communalities exceed 1
- again the EV-ONE rule appears to be of questionable value

The Scree Plot

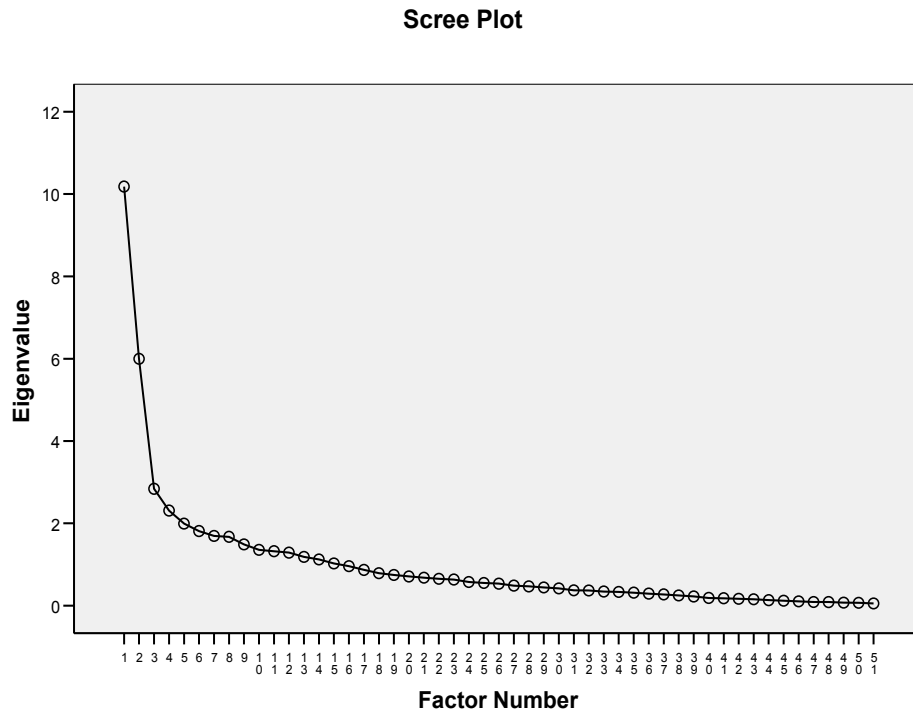


- but the scree plot suggests that 1 may be a reasonable choice for the # of factors
 - which is the recommended # of scales for CDI
- or maybe 4
 - since there is bit of a drop between 4 and 5 factors

EV-ONE Rule for DQOLY

- in SPSS, run the PAF method for the DQOLY items extracting the # of factors determined by the EV-ONE rule
 - re-enter Analyze/Data Reduction/Factor...
 - from "Variables:", replace CDI1-CDI27 by DQOLY1-DQOLY51
 - then re-execute the analysis
- converges in 14 iterations
 - but the EV-ONE rule selects 15 factors
 - seems like far too many

The Scree Plot



- the scree plot, though, suggests that 3 may be a reasonable choice for the # of factors
 - which is the recommended # of scales for DQOLY
- perhaps a somewhat larger value might also be reasonable

EV-ONE Results Summary

- the EV-ONE rule is the default approach in SPSS for choosing the # of factors
- it generated quite large choices for the # of factors for the 3 instruments of the ABC data
 - 10 for CDI, 8 for FACES, 15 for DQOLY
 - compared to recommended #'s: 1 for CDI, 2 for FACES, 3 for DQOLY
 - "it is *not* recommended, despite its wide use, because it tends to suggest too many factors" [11, p. 482]
- also rules based on % explained variance can generate much different choices for the # of factors
 - "basically inapplicable as a device to determine the # of factors" [11, p. 483]
- scree plots suggested much lower #'s of factors
 - at or close to recommended # of factors for all 3 instruments
 - but the scree plot approach is very subjective
- how many factors to extract is not simply decided

The EV-ONE Rule in SAS

Preliminary Eigenvalues: Total = 16.6895 Average = 0.55631667

	Eigenvalue	Difference	Proportion	Cumulative
1	7.96355571	5.64829078	0.4772	0.4772
2	2.31526494	0.96513775	0.1387	0.6159
3	1.35012718	0.26534277	0.0809	0.6968
4	1.08478441	0.11094797	0.0650	0.7618
5	0.97383643	0.12771977	0.0584	0.8201
6	0.84611667	0.04494166	0.0507	0.8708
7	0.80117501	0.09409689	0.0480	0.9188
8	0.70707811	0.16561419	0.0424	0.9612
9	0.54146393	0.08980179	0.0324	0.9936
10	0.45166214	0.09919908	0.0271	1.0207
11	0.35246306	0.06591199	0.0211	1.0418
12	0.28655107	0.02695638	0.0172	1.0590
13	0.25959468	0.04076514	0.0156	1.0745
14	0.21882954	0.10220557	0.0131	1.0877
15	0.11662397	0.01059049	0.0070	1.0946
16	0.10603348	0.03817049	0.0064	1.1010
17	0.06786300	0.03781255	0.0041	1.1051
18	0.03005045	0.02028424	0.0018	1.1069
19	0.00976621	0.02906814	0.0006	1.1075
20	-0.01930193	0.03167315	-0.0012	1.1063
21	-0.05097508	0.00325667	-0.0031	1.1032
22	-0.05423176	0.08408754	-0.0032	1.1000
23	-0.13831929	0.00822764	-0.0083	1.0917
24	-0.14654693	0.03833370	-0.0088	1.0829
25	-0.18488064	0.01482608	-0.0111	1.0718
26	-0.19970672	0.02597538	-0.0120	1.0599
27	-0.22568210	0.01184965	-0.0135	1.0464
28	-0.23753175	0.00463868	-0.0142	1.0321
29	-0.24217043	0.05182292	-0.0145	1.0176
30	-0.29399335		-0.0176	1.0000

4 factors will be retained by the MINEIGEN criterion.

- using the 1-step PF method in SAS
 - the EV-ONE rule is applied to eigenvalues determined from the initial communalities
 - not always to the eigenvalues from the PC's as in SPSS
- in SAS, eigenvalue-based rules can generate different choices for the # of factors when applied to different factor extraction methods
- 4 factors are generated in this case for the FACES items instead of 8 as in SPSS

SPSS Code

- SPSS is primarily a menu-driven system
 - statistical analyses are readily requested using its point and click user interface
- it does also have a programming interface
 - for more efficient execution of multiple analyses
 - with code which it calls "syntax"
 - executed in the syntax editor using the Run/All menu option
- equivalent code for a menu-driven analysis can be generated using the "paste" button
- here is code for the most recent analysis

```
FACTOR
/VARIABLES DQOLY1 TO DQOLY51 /MISSING LISTWISE
/ANALYSIS DQOLY1 TO DQOLY51 /PRINT INITIAL EXTRACTION
/PLOT EIGEN /CRITERIA MINEIGEN(1) ITERATE(200)
/EXTRACTION PAF /ROTATION NOROTATE /METHOD=CORRELATION .
```

The SAS Interface

- SAS is a menu-driven system but it starts up in its programming interface
 - statistical analyses are requested by invoking its statistical procedures or PROCs
 - PROC PRINCOMP for PCA
 - PROC FACTOR for factor analysis
- it also has a feature called Analyst for conducting menu-driven statistical analyses
 - click on Solutions/Analysis/Analyst to enter it
- but not all statistical analyses are supported
 - Analyst supports PCA but not factor analysis
- need to use the programming interface to conduct a factor analysis in SAS

SAS PROC FACTOR Code

- the following code runs the 1-step PC method with # of factors determined by the EV-ONE rule applied to the FACES items assuming they are in the default data set

```
PROC FACTOR METHOD=PRINCIPAL PRIORS=ONE MINEIGEN=1;  
  VAR FACES1-FACES30;  
RUN;
```

- to request the 1-step PC method, use "METHOD=PRINCIPAL" with "PRIORS=ONE" (i.e, set initial/prior communalities to 1)
- to request the EV-ONE rule, use "MINEIGEN=1"
- to request a specific # f of factors, replace "MINEIGEN=1" with "NFACTORS=f"
- to request the 1-step PF method, change to "PRIORS=SMC" (i.e, estimate the initial/prior communalities using the **S**quared **M**ultiple **C**orrelations)
- to iterate either of the above, change to "METHOD=PRINIT"
 - can use "MAXITER=m" to request more than the default of 30 iterations
 - adding "HEYWOOD" can avoid convergence problems

Setting the Number of Factors

- SPSS provides 2 alternatives
 - choose "Eigenvalues over:" with the default of 1 or with some other value x
 - the default is to use the EV-ONE rule
 - or choose "Number of factors:" and provide a specific integer f (no more than the # of items)
- SAS provides 3 alternatives
 - set "MINEIGEN= x " with $x=1$ to get the EV-ONE rule
 - set "NFACTORS= f " for a specific integer f
 - set "PERCENT= p " meaning the first so many factors whose combined eigenvalues explain over $p\%$ of the total variance
 - if none set, as many factors as there are items are extracted
 - if more than one set, the smallest such # is extracted

Part 3

Factor Extraction

- survey of factor extraction methods
- goodness of fit test and penalized likelihood criteria
- factoring the correlation vs. the covariance matrix
- generating factor scores
- correlation/covariance residuals
- sample size and sampling adequacy
- missing values
- example analyses

SPSS Factor Extraction Methods

- 7 different alternatives are supported in SPSS
 - principal component (1-step) + principal axis factoring (PAF)
 - PC-based factor extraction methods
 - unweighted least squares + generalized least squares
 - minimizing the sum of squared differences between the usual correlation estimates and the ones for the factor analysis model
 - with squared differences weighted in the generalized case
 - alpha factoring
 - maximizing the reliability (i.e., Chronbach's alpha) for the factors
 - maximum likelihood
 - treating the standardized items as multivariate normally distributed with factor analysis correlation structure
 - image factoring
 - Kaiser's image analysis of the image covariance matrix
 - matrix computed from the correlation matrix **R** and the diagonal elements of its inverse matrix; related to anti-image covariance matrix

SAS Factor Extraction Methods

- 9 different alternatives are supported in SAS
 - the PC and PF methods
 - with 1-step and iterated versions of both (4 PC-based methods)
 - PAF in SPSS is the same as the SAS iterated PF method
 - unweighted least squares
 - but not generalized least squares as in SPSS
 - alpha factoring
 - maximum likelihood
 - image component analysis
 - applying the PC method to the image covariance matrix
 - not the same as image factoring in SPSS but both use the image covariance matrix
 - Harris component analysis
 - uses a matrix computed from the correlation and covariance matrices
- the results for some methods can be affected by how the initial communalities are estimated

Factor Extraction Alternatives

- have demonstrated so far
 - PC method
 - PF method
- will now demonstrate
 - alpha factoring
 - maximum likelihood (ML)
- this covers the more commonly used methods [1,12]
- will not demonstrate other available methods
 - described as lesser-used in [13,p.362]

Chronbach's Alpha (α)

- a measure of internal consistency reliability
 - α is computed for each scale of an instrument separately
 - after reverse coding items when appropriate
 - by convention, an acceptable value is one that is at least .7 [12]
- α is often the only quantity used to assess established scales, and so it seems desirable for scales to have maximum α

Alpha Factoring Example

- in SPSS, run the alpha factoring method for the FACES items extracting the recommended # of 2 factors
 - re-enter Analyze/Data Reduction/Factor...
 - set "Variables:" to FACES1-FACES30
 - in "Extraction...", set "Method:" to "Alpha factoring", select "Numbers of Factors:" and set it to 2
 - then re-execute the analysis

Loadings

Factor Matrix^a

	Factor	
	1	2
FACES1	.672	.075
FACES2	.582	-.016
FACES3	-.289	.423
FACES4	.465	.164
FACES5	.526	.079
FACES6	.335	.279
FACES7	.683	-.012
FACES8	.794	-.022
FACES9	-.265	.367
FACES10	.312	.384
FACES11	.364	.276
FACES12	-.292	.123
FACES13	.268	.130
FACES14	.204	.406
FACES15	-.224	.566
FACES16	.592	.162
FACES17	.652	-.015
FACES18	.676	.215
FACES19	-.458	.402
FACES20	.546	.130
FACES21	.518	.112
FACES22	.649	.004
FACES23	.663	-.172
FACES24	-.298	.236
FACES25	-.514	.504
FACES26	.705	.017
FACES27	.521	.190
FACES28	-.245	.352
FACES29	-.474	.302
FACES30	.610	.152

Extraction Method: Alpha Factoring.

a. 2 factors extracted. 7 iterations required.

- the matrix of loadings
- FACES1 once again loads much more highly on the first factor
 - since .672 is much larger than .075
 - once again the loadings have changed only a little
 - from .702 and .110 for the PC method

Problems with Alpha Factoring

- the alpha factoring method converged in only 7 iterations for 2 factors using the FACES items
- however, it does not converge for 1 or 3 factors using the FACES items
 - even with the # of iterations set to 1000
 - it seems to be cycling, never getting close to a solution
- for the CDI items, it does not converge for 1, 2, or 3 factors
- for DQOLY, it does not converge for 1 or 3 factors, but does converge for 2 factors
- the alpha factoring method seems very unreliable
- even when it works, its optimal properties are lost following rotation [11, p. 482]

Maximum Likelihood Example

- in SPSS, run the ML method for the FACES items extracting the recommended # of 2 factors
 - re-enter Analyze/Data Reduction/Factor...
 - in "Extraction...", change "Method:" to "Maximum likelihood"
 - then re-execute the analysis
- estimates the correlation matrix **R** using its most likely value given the observed data assuming **R** has factor analysis structure and that item values are normally distributed or at least approximately so [1]

Loadings

Factor Matrix^a

	Factor	
	1	2
FACES1	.692	.114
FACES2	.590	-.043
FACES3	-.328	.491
FACES4	.406	-.015
FACES5	.512	.003
FACES6	.321	.226
FACES7	.643	.026
FACES8	.769	-.004
FACES9	-.317	.230
FACES10	.314	.472
FACES11	.354	.091
FACES12	-.314	.033
FACES13	.279	.173
FACES14	.150	.240
FACES15	-.222	.426
FACES16	.614	.282
FACES17	.632	-.064
FACES18	.678	.316
FACES19	-.484	.488
FACES20	.536	.191
FACES21	.537	.157
FACES22	.644	.147
FACES23	.670	-.096
FACES24	-.298	.241
FACES25	-.538	.596
FACES26	.720	.151
FACES27	.486	.125
FACES28	-.218	.300
FACES29	-.482	.330
FACES30	.634	.235

- the matrix of loadings
- FACES1 once again loads much more highly on the first factor
 - since .692 is much larger than .114
 - the loadings have changed, but only a little
 - from .702 and .110 for the PC method
- all 4 extraction methods generate similar loadings, at least for FACES1

Extraction Method: Maximum Likelihood.

a. 2 factors extracted. 5 iterations required.

Goodness of Fit Test

- for the ML method, it is possible to test how well the factor analysis model fits the data

H_0 : the correlation matrix **R** equals the one based on 2 factors vs.

H_a : it does not

- p-value = .000 < .05 is significant so reject H_0 , but would like not to reject

Goodness-of-fit Test		
Chi-Square	df	Sig.
572.052	376	.000

- can search for the first # of factors for which this test becomes nonsignificant

- significant for 7 factors
- nonsignificant for 8 factors
- but this is not close to the recommended # of 2 factors

Goodness-of-fit Test		
Chi-Square	df	Sig.
290.767	246	.026

Goodness-of-fit Test		
Chi-Square	df	Sig.
250.667	223	.098

Maximum Likelihood in SAS

- get the same loadings as for SPSS
 - use "METHOD=ML" with "PRIORS=SMC" (to estimate the initial/prior communalities using the squared multiple correlations)
- but the goodness of fit test is replaced by a similar test
 - seems to be something like a one-sided version of the test in SPSS with alternative hypothesis that more than the current # of factors are required
 - but 8 is also the first # of factors for which this test is nonsignificant (but at $p=.0894$ compared to $p=.098$ in SPSS)

Test	DF	Chi-Square	ChiSq
H0: 8 Factors are sufficient	223	251.8939	0.0894
HA: More factors are needed			

- in any case, this test tends to generate "more factors than are practical" [11,p. 479]

Penalized Likelihood Criteria

- SAS generates 2 penalized likelihood criteria
 - for selecting between alternative models
 - models with more parameters have larger likelihoods, so offset this with more of a penalty for more parameters
 - and transform so that smaller values indicate better models
 - AIC (Akaike's Information Criterion)
 - penalty based on the # of parameters
 - BIC (Schwarz's Bayesian Information Criterion)
 - penalty based on the # of observations/cases as well as the # of parameters
 - neither are available in SPSS
 - the AIC option in SPSS syntax requests display of the anti-image covariance matrix⁷⁶

Results for AIC/BIC

- the following are the values for k=8 factors

Akaike's Information Criterion	-146.66197
Schwarz's Bayesian Criterion	-734.20653

- an AIC (BIC) value does not mean anything by itself
 - it needs to be compared to AIC (BIC) values for other models
- the minimum AIC is achieved at 9 factors
 - seems too large
 - "AIC tends to include factors that are statistically significant but inconsequential for practical purposes" [14, p. 1336]
- the minimum BIC is achieved at 2 factors
 - the only approach so far to select the recommended # of factors
 - "seems to be less inclined to include trivial factors" [14, p. 1336]

The Matrix Being Factored

- by default, SPSS/SAS factor the correlation matrix $\mathbf{R}$
 - factoring the standardized items z
 - for y 's, subtract means, divide by standard deviations, then factor
 - the most commonly used approach
- both have an option to factor the covariance matrix Σ
 - in SPSS, click on "Covariance matrix" in "Extraction..."
 - in SAS, add "COVARIANCE" to PROC FACTOR statement
 - factoring the centered items instead
 - for y 's, subtract means, then factor
 - so the total variance is now the sum of the variances for all the items and the EV-ONE rule should not be used
 - only works with some factor extraction methods
- SAS also allows factoring without subtracting means
 - with or without dividing y 's by standard deviations
 - add "NOINT" to PROC FACTOR statement

Factoring a Covariance Matrix

- in SPSS, run the PAF method on the covariance matrix for the FACES items extracting the recommended # of 2 factors
 - re-enter Analyze/Data Reduction/Factor...
 - in "Extraction...", change "Method:" to "Principal axis factoring" and turn on "Covariance matrix"
 - then re-execute the analysis
- SPSS generates 2 types of output
 - "raw" output is for the (raw) covariance matrix
 - "rescaled" output is for the correlation matrix obtained by rescaling results for the covariance matrix
 - in SAS, "weighted" is the same as "raw" in SPSS (i.e., the covariance matrix is a weighted correlation matrix) while "unweighted" is the same as "rescaled"
 - the SPSS/SAS manuals do not provide details on factoring a covariance matrix, so the above is a best guess

Loadings

Factor Matrix^a

	Raw		Rescaled	
	Factor		Factor	
	1	2	1	2
FACES1	.585	.094	.679	.109
FACES2	.610	-.035	.585	-.034
FACES3	-.381	.611	-.319	.511
FACES4	.515	.087	.445	.075
FACES5	.654	.063	.535	.051
FACES6	.447	.363	.334	.271
FACES7	.700	.004	.659	.004
FACES8	.878	-.020	.787	-.018
FACES9	-.349	.372	-.295	.315
FACES10	.400	.567	.318	.451
FACES11	.426	.220	.369	.191
FACES12	-.313	.068	-.304	.066
FACES13	.311	.158	.278	.141
FACES14	.185	.353	.173	.330
FACES15	-.250	.533	-.226	.483
FACES16	.640	.254	.604	.240
FACES17	.654	-.033	.625	-.031
FACES18	.704	.271	.659	.253
FACES19	-.580	.551	-.471	.447
FACES20	.579	.201	.532	.184
FACES21	.492	.124	.531	.134
FACES22	.710	.108	.650	.099
FACES23	.660	-.141	.666	-.143
FACES24	-.366	.294	-.304	.243
FACES25	-.556	.583	-.530	.555
FACES26	.739	.103	.707	.098
FACES27	.585	.172	.508	.150
FACES28	-.250	.363	-.217	.314
FACES29	-.500	.332	-.474	.315
FACES30	.673	.231	.618	.212

Extraction Method: Principal Axis Factoring.
a. 2 factors extracted. 6 iterations required.

- the matrix of loadings
 - use the rescaled loadings to be consistent with prior analyses
 - these are the only ones reported by SAS
- FACES1 once again loads much more highly on the first factor
 - since .679 is much larger than .109
 - the loadings have changed, but only a little
 - from .683 and .107 for the PAF method applied to the correlation matrix
- does not appear to be much of an impact to factoring the covariance matrix vs. the correlation matrix

Generating the Factor Scores

- factors identified by factor analysis have construct validity if they predict certain related variables
- this can be assessed using the factor scores which are estimates of the values of the factors for each of the observations/cases in the data set
- first generate factor score variables
 - In SPSS, click on "Scores..." and turn on "Save as variables"
 - variables are added at the end of the data set called FAC1_1, FAC2_1, etc.
 - in SAS, add the "SCORE" option to the PROC FACTOR statement and specify a new data set name using the "OUT=" option
 - a new data set is created with the specified name containing everything in the source data set plus variables called Factor1, Factor2, etc.
- then use these variables as predictors in regression models of appropriate outcome variables

Correlation Residuals

- how much correlations generated by the factor analysis model differ from standard estimates of the correlations
 - measures how well the model fits correlations between items
 - when the covariance matrix is factored, covariance residuals are generated instead
 - to generate correlation residuals in SAS
 - add the "RESIDUALS" option to the PROC FACTOR statement to generate listings of these residuals
 - further adding the "OUTSTAT=" option gives a name to an output data set containing among other things the correlation residuals for further analysis
 - in SPSS, use "Reproduced" for the "Correlation matrix" option of "Descriptives..." to generate a listing of residuals
- these do not directly address the issue of whether the values for the items are reasonably treated as close to normally distributed or if any are outlying
 - item residuals address this issue
 - such results are reported later

Sample Size Considerations

- sample sizes for planned factor analyses are based on conventional guidelines
 - not on formal power analyses
 - recommendations for the sample size vary from 3 to 10 times the # of items and at least 100 [8,13,14]
 - higher values seem more important for development of new scales than for assessment of established scales
 - for the ABC data, there are only 3.8, 3.4, and 2.0 observations per item for the CDI, FACES, and DQOLY items, respectively
 - relatively low values especially for DQOLY

Measure of Sampling Adequacy

- possible to assess the sampling adequacy of existing data
 - using the Kaiser-Meier-Olkin (KMO) measure of sampling adequacy (MSA)
 - a summary of how small partial correlations are relative to ordinary correlations
 - values at least .8 are considered good
 - values under .5 are considered unacceptable
 - in SPSS, click on "Descriptives..." and set "KMO and Bartlett's test of sphericity" on
 - in SAS, add the "MSA" option to the PROC FACTOR statement
 - calculates overall MSA value + MSA values for each item
 - also get Bartlett's test of sphericity in SPSS
 - in SAS, it is only generated for the ML method
- H_0 : the standardized items are independent (0 factor model)
- H_a : they are not (i.e., there is at least 1 factor)

Results for the ABC Data

- observed sampling adequacy
 - .778 for FACES
 - .725 for CDI
 - .699 for DQOLY
 - ABC items are somewhat adequate ($>.5$) but not good ($<.8$)
- Bartlett's test of sphericity

H_0 : independent standardized items

H_a : they are not

 - $p = .000$ for all 3 cases
 - all three sets of standardized items are distinctly correlated and so require at least 1 factor
 - however, this test is not considered of value [11, p.469]

FACES

KMO and Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.778
Bartlett's Test of Sphericity	Approx. Chi-Square	1365.068
	df	435
	Sig.	.000

CDI

KMO and Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.725
Bartlett's Test of Sphericity	Approx. Chi-Square	920.324
	df	351
	Sig.	.000

DQOLY

KMO and Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.699
Bartlett's Test of Sphericity	Approx. Chi-Square	2911.235
	df	1275
	Sig.	.000

Missing Values

- by default, SPSS (SAS) deletes any cases (observations) with missing values for any of the items
- SPSS supports
 - "Exclude cases listwise", the default option
 - "Exclude cases pairwise"
 - calculating correlations between pairs of items using all cases with non-missing values for both items
 - can generate very unreliable estimates so best not to use
 - "Replace with mean"
 - replace missing values for an item with the average of all the non-missing values for that item
- SAS provides no other options
 - but can first impute values using PROC MI (for multiple imputation)

Missing Item Value Imputation

- many instruments do not provide missing value guidelines
- when they do, they usually suggest replacing missing item values with averages of the non-missing item values for a case
 - averaging values of the other items for that case rather than values of the other cases for that item
 - so different from the SPSS "Replace with mean" option
- as long as there aren't too many items with missing values for that case
 - e.g., if at least 50% or 70% of the item values are not missing

Part 4

Factor Rotation

- marker items, allocating items to factors/scales, discarding items
- varimax rotation, normalization, testing for significant loadings
- orthogonal vs. oblique rotations, survey of alternative rotation approaches
- promax rotation, inter-factor correlations, the structure matrix
- impact of rotations
- reverse coding
- example analyses

Marker Items for Factors

- item z is considered a marker item (or a salient) for factor F if its absolute loading is high while its absolute loadings on all the other factors F' are all low
 - the absolute loading is the loading with its sign removed
 - when discussing this, authors often ignore the issue of negative loadings, but in general signs of loadings need to be accounted for
- what is meant by high?
 - typically, an absolute loading at or above a cutoff value, like 0.3, 0.35, 0.4, or 0.5 [8,15,16], is considered high while anything below that is considered low
 - at least 0.3 at a minimum; at least 0.5 usually better [11]
- if some factors have small #'s of marker items, the # of factors may have been set too high
 - at least 2 [11] or 3 [8,13] items per factor is desirable

Item-Scale Allocation

- when developing scales for a new instrument, the items are usually separated into disjoint sets consisting of the marker items for each factor and used to compute associated scales
 - marker items represent distinct aspects of associated factors
 - and are the basis for assigning scales meaningful names
- items that have high absolute loadings on more than one factor are usually discarded [8]
 - they do not represent distinct aspects of only one factor
- items that have low absolute loadings on all factors should then also be discarded
 - they do not represent distinct aspects of any factor
 - most authors ignore this issue, but it does happen quite often in practice

General vs. Group Factors

- should all items load on all factors or not?
 - general factors are those with all items loading on them
 - this is assumed in the standard factor analysis model
 - group factors are those with associated subsets of items loading on them
 - this is the basis for item-scale allocation rules
 - "not everyone agrees that general factors are undesirable"
[11, p. 503]
- instruments which partition their items into disjoint sets corresponding to marker items are assuming that all the factors are distinct group factors
- instruments that use all items to compute all the scales are assuming the factors are all general factors
 - e.g., the PCS and MCS scales of the MOS SF-36 are computed from all 35 items used in scale construction
 - but these items are first partitioned into disjoint groups and used to compute associated subscales

Rotation

- the interpretation of factors through their marker items can be difficult if based on the loadings generated directly by factor extraction
- rotated loadings are typically used instead
 - these are thought to be more readily interpretable
- varimax is the most popular approach [8,12]
 - it attempts to minimize the # of z's that load highly on each of the factors
- but there are a variety of other ways to rotate loadings

Varimax Rotation for FACES

- in SPSS, run the ML method for the FACES items extracting the recommended # of 2 factors and rotate loadings using varimax rotation
 - re-enter Analyze/Data Reduction/Factor...
 - in "Extraction...", change "Method:" to "Maximum likelihood"
 - note there is no option for which type of matrix to factor
 - it does not matter for ML
 - the ML estimate of the correlation matrix induces the ML estimate of the covariance matrix and vice versa
 - in "Rotation...", click on "Varimax"
 - note the default rotation setting is "None"
 - then re-execute the analysis

Rotating the Initial Loadings

Factor Matrix^a

	Factor	
	1	2
FACES1	.692	.114
FACES2	.590	-.043
FACES3	-.328	.491
FACES4	.406	-.015
FACES5	.512	.003
FACES6	.321	.226
FACES7	.643	.026
FACES8	.769	-.004
FACES9	-.317	.230
FACES10	.314	.472
FACES11	.354	.091
FACES12	-.314	.033
FACES13	.279	.173
FACES14	.150	.240
FACES15	-.222	.426
FACES16	.614	.282
FACES17	.632	-.064
FACES18	.678	.316
FACES19	-.484	.488
FACES20	.536	.191
FACES21	.537	.157
FACES22	.644	.147
FACES23	.670	-.096
FACES24	-.298	.241
FACES25	-.538	.596
FACES26	.720	.151
FACES27	.486	.125
FACES28	-.218	.300
FACES29	-.482	.330
FACES30	.634	.235

Extraction Method: Maximum Likelihood.

a. 2 factors extracted. 5 iterations required.

- the matrix of loadings
 - with 30 rows and 2 columns
- is multiplied on the right by the factor transformation matrix
 - with 2 rows and 2 columns
 - the one below is produced by varimax
- to produce the rotated factor matrix
 - will also have 30 rows and 2 columns
- same process for any rotation scheme but using a different transformation matrix

Factor Transformation Matrix

Factor	1	2
1	.844	-.536
2	.536	.844

Extraction Method: Maximum Likelihood.

Rotation Method: Varimax with Kaiser Normalization.

Varimax Rotated Loadings

Rotated Factor Matrix^a

	Factor	
	1	2
FACES1	.645	-.274
FACES2	.475	-.353
FACES3	-.014	.591
FACES4	.335	-.230
FACES5	.434	-.272
FACES6	.392	.018
FACES7	.557	-.322
FACES8	.647	-.415
FACES9	-.144	.364
FACES10	.518	.230
FACES11	.347	-.113
FACES12	-.247	.196
FACES13	.329	-.004
FACES14	.255	.123
FACES15	.041	.478
FACES16	.669	-.091
FACES17	.499	-.393
FACES18	.742	-.096
FACES19	-.146	.671
FACES20	.555	-.126
FACES21	.538	-.156
FACES22	.623	-.221
FACES23	.514	-.440
FACES24	-.122	.363
FACES25	-.134	.792
FACES26	.688	-.259
FACES27	.477	-.155
FACES28	-.024	.370
FACES29	-.230	.536
FACES30	.661	-.142

- the matrix of rotated loadings
 - with 30 rows and 2 columns
- FACES1 once again loads more highly on the first factor
 - since .645 is larger than .274
 - loadings have changed quite a bit
 - from .702 and .110 for the PC method
 - especially the loading (–.245) on factor 2 which is now negative

Extraction Method: Maximum Likelihood.

Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 3 iterations.

Reallocated Explained Variance

- the same percentage of total variance (32.814%) is explained using rotated loadings as with unrotated loadings
- but it is allocated differently to the factors
 - factor 2's contribution has increased from 6.937% to 12.107%
 - while factor 1's contribution has decreased from 25.877% to 20.707%

Total Variance Explained

Factor	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	8.360	27.867	27.867	7.763	25.877	25.877	6.212	20.707	20.707
2	2.777	9.255	37.122	2.081	6.937	32.814	3.632	12.107	32.814
3	1.804	6.012	43.134						
4	1.593	5.309	48.443						
5	1.413	4.712	53.155						
6	1.305	4.350	57.505						
7	1.266	4.221	61.726						
8	1.150	3.835	65.560						
9	.984	3.279	68.839						
10	.898	2.992	71.831						
11	.818	2.726	74.557						
12	.770	2.567	77.124						
13	.708	2.359	79.484						
14	.681	2.268	81.752						
15	.583	1.945	83.697						
16	.563	1.876	85.573						
17	.519	1.731	87.304						
18	.481	1.604	88.908						
19	.453	1.509	90.417						
20	.407	1.357	91.774						
21	.381	1.270	93.043						
22	.361	1.204	94.248						
23	.310	1.035	95.282						
24	.280	.933	96.215						
25	.251	.836	97.051						
26	.226	.752	97.803						
27	.209	.697	98.500						
28	.192	.641	99.141						
29	.155	.516	99.657						
30	.103	.343	100.000						

Extraction Method: Maximum Likelihood.

Sorting the Rotated Loadings

- to more readily allocate items to factors, have items displayed in sorted order based on their loadings
 - in SPSS
 - re-enter Analyze/Data Reduction/Factor...
 - in "Options...", click on "Sorted by size"
 - then re-execute the analysis
 - in SAS
 - add the "REORDER" option to the PROC FACTOR statement

Sorted Rotated Loadings

Rotated Factor Matrix^a

	Factor	
	1	2
FACES18	.742	-.096
FACES26	.688	-.259
FACES16	.669	-.091
FACES30	.661	-.142
FACES8	.647	-.415
FACES1	.645	-.274
FACES22	.623	-.221
FACES7	.557	-.322
FACES20	.555	-.126
FACES21	.538	-.156
FACES10	.518	.230
FACES23	.514	-.440
FACES17	.499	-.393
FACES27	.477	-.155
FACES2	.475	-.353
FACES5	.434	-.272
FACES6	.392	.018
FACES11	.347	-.113
FACES4	.335	-.230
FACES13	.329	-.004
FACES14	.255	.123
FACES12	-.247	.196
FACES25	-.134	.792
FACES19	-.146	.671
FACES3	-.014	.591
FACES29	-.230	.536
FACES15	.041	.478
FACES28	-.024	.370
FACES9	-.144	.364
FACES24	-.122	.363

Extraction Method: Maximum Likelihood.

Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 3 iterations.

- column 1 values decrease in absolute value from FACES18 to FACES12 while remaining larger in absolute value than column 2 values
 - so 22 load more on factor 1: 18,28,...,12
- after that column 2 values decrease in absolute value while remaining larger in absolute value than column 1 values
 - other 8 load more on factor 2: 25,19,3,29,15,28,9,24
- need to know what the items are in order to interpret these results
- item 12 is the only item with maximum absolute loading for a negative loading
 - suggesting it will need reverse coding

Discarding Items

Rotated Factor Matrix^a

	Factor	
	1	2
FACES18	.742	-.096
FACES26	.688	-.259
FACES16	.669	-.091
FACES30	.661	-.142
FACES8	.647	-.415
FACES1	.645	-.274
FACES22	.623	-.221
FACES7	.557	-.322
FACES20	.555	-.126
FACES21	.538	-.156
FACES10	.518	.230
FACES23	.514	-.440
FACES17	.499	-.393
FACES27	.477	-.155
FACES2	.475	-.353
FACES5	.434	-.272
FACES6	.392	.018
FACES11	.347	-.113
FACES4	.335	-.230
FACES13	.329	-.004
FACES14	.255	.123
FACES12	-.247	.196
FACES25	-.134	.792
FACES19	-.146	.671
FACES3	-.014	.591
FACES29	-.230	.536
FACES15	.041	.478
FACES28	-.024	.370
FACES9	-.144	.364
FACES24	-.122	.363

Extraction Method: Maximum Likelihood.

Rotation Method: Varimax with Kaiser Normalization.

a. Rotation converged in 3 iterations.

- using 0.3 as cutoff for low/high loadings
 - items 8,7,23,17,2 have both absolute loadings > 0.3
 - items 14,12 have both absolute loadings < 0.3
 - suggests discarding 7 items
- using 0.4 instead
 - only items 8,23 now have both loadings high
 - but items 6,11,4,13,14,12,28,9,24 now have both loadings low
 - suggests discarding 11 items
- FACES is an established instrument so it seems inappropriate to discard its items
 - but if so many items of an established instrument can be considered of negligible value, perhaps meaningful items can be discarded when developing new scales

Normalizing Before Rotating

- by default, SPSS/SAS normalize the factor matrix prior to rotating it to reduce computational problems
 - both use Kaiser normalization as the default
 - dividing each row of the factor matrix by the sum of squares of the values in the row
- SPSS also supports the case of no normalization
 - but this can only be selected using the programming interface, not with the menu-driven interface
- SAS supports requests for the following
 - Kaiser normalization
 - no normalization
 - the Cureton-Mulaik weighting technique
 - rescaling rows to represent covariances rather than correlations

Testing Loadings in SAS

Rotated Factor Pattern
With 95% confidence limits; Cover 0?
Estimate/StdErr/LowerCL/UpperCL/Coverage Display

		Factor1	Factor2
FACES1	FACES1	0.64504	-0.27420
		0.06450	0.09932
		0.50072	-0.45570
		0.75446	-0.07080
		0[]	[]0
...			
FACES30	FACES30	0.66117	-0.14169
		0.06204	0.10117
		0.52184	-0.33193
		0.76614	0.05963
		0[]	[0]

- FACES1 loads on both factors
- FACES30 loads on factor 1 but not factor 2

- SAS has an option to test for significantly nonzero loadings
 - add "COVER" to test if loadings equal zero or not
 - "COVER=p" tests for loadings equal to p
 - by default p=0
 - "[0]" means 0 is in the 95% confidence interval for a loading
 - "0[]" means it is not

Significant Loadings for FACES

- significant rotated loadings (using varimax rotation)
 - 0 items load on neither factor
 - all FACES items appear to be of distinct value
 - 11 items load only on factor 1 and 7 only on factor 2
 - 12 items load on both factors, so 40% of the items address both factors
 - adaptability and cohesion are likely to be highly correlated
- comparison to the recommended scales
 - adaptability based on 14 even items other than item 30
 - 12 of these load highly on factor 1
 - item 24,28 load highly only on factor 2
 - cohesion based on the 15 odd items + item 30
 - 11 of these load highly on factor 2
 - items 11,13,21,27,30 load highly only on factor 1
- identified factors appear to be distinctly different from standard FACES constructs with 23.3% (7/30) inconsistent items

Varimax Rotation for DQOLY

- in SPSS, run the ML method for the DQOLY items extracting the recommended # of 3 factors and rotate loadings using varimax rotation
 - re-enter Analyze/Data Reduction/Factor...
 - change "Variables:" to DQOLY1-DQOLY51
 - in "Extraction..." change "Number of factors:" to 3
 - then re-execute the analysis
- will not consider rotations of CD1-CDI27 since the recommended # of scales is 1 and so rotations are unnecessary

Using Sorted Rotated Loadings

- assigning items to factors
 - 18 load more on factor 1: 35-51, 7
 - 26 load more on factor 2: 1-6, 8-23, 31-34
 - 7 load more on factor 3: 24-30
 - need to know what the items are in order to interpret these results
- items that are possibly discardable
 - using 0.3 as the cutoff for low/high
 - 40,34 load on more than 1 factor while 7,15,16 load on 0 factors
 - suggests discarding 5 items
 - using 0.4 as the cutoff for low/high
 - 0 items load on > 1 factor, but 10 load on no factors: 7,8,9,19,17,14,31,12,15,16
 - suggests discarding 10 items

Significant Loadings for DQOLY

significant rotated loadings (using varimax rotation)

- 0 items load on 0 factors
 - all DQOLY items appear to be of distinct value
- 30 items load on exactly 1 factor
- 19 items load on exactly 2 factors, 3 on all 3 factors
 - so 43% of the items address multiple factors
- comparison to recommended scales
 - of the 17 satisfaction items (35-51), all load on factor 1
 - of the 23 impact items (1-23), all but 1 load on factor 2
 - all but item 7
 - of the 11 worry items (24-34), all but 2 load on factor 3
 - all but items 31,32
 - identified factors appear to be similar to standard DQOLY constructs with only 5.9% (3/51) inconsistent items

Significant Loadings for CDI

- using ML factor extraction of 1 factor without rotation and with the COVER option in SAS
- significant unrotated loadings
 - 4 items do not load on the 1 factor
 - items 9,23,25,26
 - 23 items load on the 1 factor
- a substantial amount of 17.4% (4/27) of the items appear to be of negligible value for the ABC subjects

Orthogonal vs. Oblique Rotations

- factors are independent of each other under the factor analysis model satisfying

$$z=L(1)\cdot F(1)+L(2)\cdot F(2)+\dots+L(k)\cdot F(k)+u$$

- rotations change both the loadings and the factors in such a way that the same relationships hold as before

$$z=L'(1)\cdot F'(1)+L'(2)\cdot F'(2)+\dots+L'(k)\cdot F'(k)+u$$

- an orthogonal rotation preserves perpendicularity between the axes
 - which means that factors remain independent
- an oblique rotation does not preserve perpendicularity between the axes
 - which means that factors become correlated

SPSS Rotation Approaches

- the default rotation approach is not to rotate ("None")
- 3 orthogonal rotation approaches are supported
 - Varimax, Quartimax, Equamax
- 2 oblique rotation approaches are supported
 - Direct Oblimin
 - changes with parameter called Delta with default value 0
 - becomes less oblique as Delta becomes more negative
 - Promax
 - starts with a Varimax rotation
 - changes with parameter called Kappa with default value 4

SAS Rotation Approaches

- the default rotation approach is not to rotate
 - use "ROTATE=" option to assign a rotation scheme, e.g., "ROTATE=VARIMAX"
- many orthogonal rotation approaches are supported
 - ORTHOMAX with a weight parameter called GAMMA
 - GAMMA=1 by default, same as VARIMAX
 - GAMMA=0, same as QUARTIMAX
 - GAMMA=(# of factors)/2, same as EQUAMAX
 - GAMMA=.5, same as BIQUARTIMAX
 - GAMMA=(# of items), same as FACTORPARSIMAX
 - GAMMA=(# of items)·(# of factors – 1)/(# of items + # of factors – 2), same as PARSIMAX
 - these include all orthogonal approaches supported in SPSS
 - orthogonal Crawford-Ferguson rotation approaches
 - ORTHCF with 2 parameters, ORTHGENCF with 4 parameters

SAS Rotation Approaches

- many oblique rotation approaches are also supported
 - OBLIMIN with a weight parameter called TAU
 - TAU=0 is same default as in SPSS (but called Delta), same as QUARTIMIN
 - TAU=1, same as COVARIMIN
 - TAU=.5, same as BIQUARTIMIN
 - PROMAX with a parameter called POWER
 - POWER=3 is the default rather than 4 as in SPSS (but called Kappa)
 - by default, it starts with a VARIMAX orthogonal rotation as in SPSS, but can be started from any other orthogonal or oblique rotation
 - Harris-Kaiser (HK) with a parameter called HKPOWER having default of 0.0
 - when HKPOWER=1, HK becomes VARIMAX
 - Harris-Kaiser type oblique versions of other orthogonal approaches are also possible
 - oblique versions of all orthogonal approaches are available
 - but not clear if they overlap with the above or not
 - includes all orthogonal approaches supported in SPSS

Oblique Rotation Example

- in SPSS, run the ML method for the FACES items extracting the recommended # of 2 factors and rotate loadings using promax rotation with its default parameter setting
 - re-enter Analyze/Data Reduction/Factor...
 - change "Variables:" to FACES1-FACES30
 - in "Extraction..." change "Number of factors:" to 2
 - in "Rotation...", click on "Promax"
 - leave Kappa at its default value of 4
 - to get the same result as the default promax rotation in SAS, change Kappa to 3
 - then re-execute the analysis

Rotating the Initial Loadings

Factor Matrix^a

	Factor	
	1	2
FACES1	.692	.114
FACES2	.590	-.043
FACES3	-.328	.491
FACES4	.406	-.015
FACES5	.512	.003
FACES6	.321	.226
FACES7	.643	.026
FACES8	.769	-.004
FACES9	-.317	.230
FACES10	.314	.472
FACES11	.354	.091
FACES12	-.314	.033
FACES13	.279	.173
FACES14	.150	.240
FACES15	-.222	.426
FACES16	.614	.282
FACES17	.632	-.064
FACES18	.678	.316
FACES19	-.484	.488
FACES20	.536	.191
FACES21	.537	.157
FACES22	.644	.147
FACES23	.670	-.096
FACES24	-.298	.241
FACES25	-.538	.596
FACES26	.720	.151
FACES27	.486	.125
FACES28	-.218	.300
FACES29	-.482	.330
FACES30	.634	.235

Extraction Method: Maximum Likelihood.

a. 2 factors extracted. 5 iterations required.

- the matrix of loadings
 - with 30 rows and 2 columns
- it is multiplied on the right by the factor transformation matrix for a varimax rotation
 - with 2 rows and 2 columns
 - to produce the varimax-rotated factor matrix
- then this is multiplied on the right by another transformation matrix to generate the promax rotated loadings
 - will also have 30 rows and 2 columns

Promax Rotated Loadings

Pattern Matrix^a

	Factor	
	1	2
FACES1	.643	-.102
FACES2	.429	-.244
FACES3	.161	.657
FACES4	.308	-.151
FACES5	.406	-.166
FACES6	.447	.145
FACES7	.530	-.184
FACES8	.603	-.259
FACES9	-.053	.362
FACES10	.651	.422
FACES11	.357	-.016
FACES12	-.220	.141
FACES13	.368	.100
FACES14	.323	.218
FACES15	.188	.548
FACES16	.725	.111
FACES17	.444	-.281
FACES18	.806	.128
FACES19	.036	.705
FACES20	.586	.035
FACES21	.558	-.003
FACES22	.634	-.049
FACES23	.447	-.329
FACES24	-.029	.367
FACES25	.085	.843
FACES26	.697	-.071
FACES27	.491	-.022
FACES28	.084	.406
FACES29	-.098	.527
FACES30	.701	.052

- the matrix of promax rotated loadings
 - called the pattern matrix in SPSS
 - with 30 rows and 2 columns
- FACES1 once again loads more highly on the first factor
 - since .643 is larger than .102
 - with more of a difference
 - vs. .645 and -.274 for varimax
 - the first loading is about the same while the second has gotten smaller in absolute value

Extraction Method: Maximum Likelihood.

Rotation Method: Promax with Kaiser Normaliz

a. Rotation converged in 3 iterations.

Inter-Factor Correlations

- since promax is an oblique rotation, the associated factors are correlated
- the factor correlation matrix contains those correlations
 - only 1 in this case because there are only 2 factors
- the 2 factors for this case are distinctly inversely related with an estimated correlation of $-.511$

Factor Correlation Matrix

Factor	1	2
1	1.000	-.511
2	-.511	1.000

Extraction Method: Maximum Likelihood.

Rotation Method: Promax with Kaiser Normalization.

The Structure Matrix

Structure Matrix

	Factor	
	1	2
FACES1	.695	-.431
FACES2	.554	-.463
FACES3	-.175	.574
FACES4	.385	-.308
FACES5	.491	-.374
FACES6	.372	-.083
FACES7	.624	-.454
FACES8	.736	-.567
FACES9	-.238	.389
FACES10	.435	.089
FACES11	.365	-.198
FACES12	-.292	.253
FACES13	.317	-.088
FACES14	.212	.053
FACES15	-.092	.452
FACES16	.668	-.260
FACES17	.587	-.508
FACES18	.740	-.283
FACES19	-.325	.687
FACES20	.568	-.264
FACES21	.560	-.288
FACES22	.659	-.373
FACES23	.615	-.557
FACES24	-.217	.382
FACES25	-.346	.800
FACES26	.733	-.427
FACES27	.502	-.272
FACES28	-.124	.364
FACES29	-.368	.577
FACES30	.675	-.307

Extraction Method: Maximum Likelihood
Rotation Method: Promax with Kaiser Nc

- SPSS also generates a matrix it calls the structure matrix
 - SAS calls it the factor structure matrix
 - it equals the pattern matrix multiplied on the right by the factor correlation matrix
 - its entries are the correlations between the items and the factors
- the correlation between
 - FACES1 and factor 1 is .695
 - about the same as the loading of .643
 - FACES1 and factor 2 is -.431
 - much different from the loading of -.102
 - a low absolute loading may be associated with a substantially stronger correlation

The Reference Structure Matrix

- pattern matrices can have absolute loadings larger than 1 and possibly much larger [8]
 - this did not happen for this analysis
- SAS generates a reference structure matrix for use in place of the pattern matrix when it has such anomalous values
 - it is interpreted in the same way as a pattern matrix
 - not generated in SPSS
- however, if such problems occur, maybe that is an indication that the rotation approach needs to be changed

Using Sorted Rotated Loadings

- factor 1 loadings decrease in absolute value from FACES18 to FACES12 while remaining larger in absolute value than factor 2 loadings
 - 22 load more on factor 1: 18,28,...,12
- after that factor 2 loadings decrease in absolute value while remaining larger in absolute value than factor 1 loadings
 - 8 load more on factor 2: 25,19,3,29,15,28,9,24
- exactly the same as for varimax rotation
 - there no impact to using promax based on varimax over varimax without adjustment
- "orthogonal rotations usually lead one to essentially the same major groupings as oblique rotations" [11, p. 536]

Impact of Rotations

- considered 10 rotations plus no rotation [10]
 - 4 orthogonal
 - varimax, quartimax, equamax, parsimax
 - 6 oblique
 - Harris-Kaiser
 - promax starting from each of the other 5
 - with the default parameter POWER=3
 - ran this in SAS
- generated associated item-scale allocations
 - with each item allocated to the factor/scale for which it achieves its maximum absolute loading, without discarding any items

Impact of Rotations

- for the FACES items
 - all 10 rotations generated the same allocation
- for the DQOLY items
 - the 10 rotations generated 4 different allocations but these were not too different from each other
- for both the FACES and DQOLY items
 - the allocations based on unrotated loadings were much different from the ones based on rotations and from recommended allocations
- rotating the loadings appears to have a distinct impact on the results compared to not rotating them, but the choice of the rotation may not have much of an impact on those results

The Standard CDI Scale

Factor Matrix

	Factor
	1
cdi1	.611
cdi2	-.671
cdi3	.644
cdi4	.353
cdi5	-.209
cdi6	.539
cdi7	-.741
cdi8	-.295
cdi9	.172
cdi10	-.641
cdi11	-.795
cdi12	.283
cdi13	-.313
cdi14	.655
cdi15	-.258
cdi16	-.255
cdi17	.211
cdi18	-.239
cdi19	.373
cdi20	.532
cdi21	-.421
cdi22	.551
cdi23	.155
cdi24	-.463
cdi25	-.055
cdi26	.128
cdi27	.271

Extraction Method: Maximum Likelihood.

a. 1 factors extracted. 4 iterations required

- CDI has 27 items scored from 0-2
- these are summed to produce its one scale measuring the amount of depressive symptoms
- after reverse coding 13 of the items
 - items 2,5,7,8,10,11,13,15,16,18,21,24,25 are reverse coded
 - replace an item y by $2 - y$
- if 1 factor is extracted using the ML method, 13 items have negative loadings
 - the same items as are reverse coded in the standard CDI scale

The Standard FACES Scales

Factor Matrix^a

	Factor 1
FACES1	.690
FACES2	.587
FACES3	-.291
FACES4	.407
FACES5	.518
FACES6	.338
FACES7	.651
FACES8	.773
FACES9	-.311
FACES10	.336
FACES11	.351
FACES12	-.311
FACES13	.296
FACES14	.167
FACES15	-.196
FACES16	.625
FACES17	.621
FACES18	.688
FACES19	-.440
FACES20	.543
FACES21	.547
FACES22	.655
FACES23	.666
FACES24	-.284
FACES25	-.481
FACES26	.725
FACES27	.497
FACES28	-.202
FACES29	-.454
FACES30	.640

Extraction Method: Maximum Likelihood.

a. 1 factors extracted. 5 iterations required

- FACES has 30 items scored from 1-5 and 2 scales
 - family adaptability is computed by summing the even items other than item 30 with 2 items 24,28 reverse coded
 - i.e., replace an item y by $6 - y$
 - family cohesion is computed by summing the odd items plus item 30 with 6 items 3,9,15,19,25,29 reverse coded
- if 1 factor is extracted using the ML method, 9 items have negative loadings
 - the same items as are reverse coded in the standard FACES scales
 - plus one other: item 12

The Standard DQOLY Scales

Factor Matrix ^a

	Factor 1
dqoly1	-.328
dqoly2	-.288
dqoly3	-.378
dqoly4	-.207
dqoly5	-.243
dqoly6	-.333
dqoly7	.398
dqoly8	-.229
dqoly9	-.305
dqoly10	-.381
dqoly11	-.264
dqoly12	-.085
dqoly13	-.403
dqoly14	-.195
dqoly15	-.299
dqoly16	-.287
dqoly17	-.391
dqoly18	-.328
dqoly19	-.300
dqoly20	-.365
dqoly21	-.244
dqoly22	-.206
dqoly23	-.105
dqoly24	-.268
dqoly25	-.248
dqoly26	-.296
dqoly27	-.262
dqoly28	-.115
dqoly29	-.392
dqoly30	-.281
dqoly31	-.386
dqoly32	-.281
dqoly33	-.288
dqoly34	-.487
dqoly35	.589
dqoly36	.464
dqoly37	.572
dqoly38	.577
dqoly39	.477
dqoly40	.569
dqoly41	.415
dqoly42	.663
dqoly43	.671
dqoly44	.610
dqoly45	.719
dqoly46	.581
dqoly47	.677
dqoly48	.704
dqoly49	.461
dqoly50	.670
dqoly51	.597

Extraction Method: Maximum Likelihood.

a. 1 factors extracted. 8 iterations required.

- DQOLY has 51 items scored 1-5 and 3 scales
 - disease impact is the sum if the first 23 items (1-23) with item 7 reverse coded
 - worry is the sum of the next 11 items (24-34) with none reverse coded
 - satisfaction is the sum of the last 17 items (35-51) with none reverse coded
 - but these have the reverse orientation to all the other items except item 7
- if 1 factor is extracted using the ML method, 18 items have positive loadings
 - the 17 satisfaction items along with item 7
 - the ones with reverse orientation in the standard DQOLY scales

Reverse Coding Summary

- signs of the 1-factor loadings appear to provide information about appropriate reverse coding
 - even when there are more than 1 underlying factors
 - to supplement related theoretical item construction considerations [16]
- for CDI and DQOLY, items were separated into those usually reverse coded vs. those usually not
- for FACES, items were separated into those usually reverse coded plus item 12 vs. the others usually not
 - 12 is the only item in the 2-factor solution with maximum absolute loading at a negative value
- FACES item 12 is used to compute family adaptability
 - "it is hard to know what the rules are in our family"
 - less clearly defined rules are supposed to mean more adaptability
- perhaps, for the ABC subjects, more clearly defined family rules allowed them more flexibility to adapt in ways that do not violate those rules

Varimax Allocation - FACES

- FACES has 30 items and 2 recommended scales
 - family adaptability computed from even items other than item 30 with items 24,28 reverse coded
 - family cohesion computed from odd items plus item 30 with items 3,9,15,19,25,29 reverse coded
- allocations based on ML extraction of 2 factors and varimax rotation
 - items 3,9,15,19,24,25,28,29 separated from the rest
 - a much different allocation than recommended
 - all items usually reverse coded are separated from all the other items
 - explains why the negative inter-factor correlation

Varimax Allocation - DQOLY

- DQOLY has 51 items and 3 recommended scales
 - disease impact computed from items 1-23 with item 7 reverse coded
 - worry computed from items 24-34 with none reverse coded
 - satisfaction computed from items 35-51 with none reverse coded, but with reverse orientation to others except item 7
- allocations based on ML extraction of 3 factors and varimax rotation
 - satisfaction items 35-51 plus item 7
 - all impact items except item 7 plus worry items 31-34
 - worry items 24-30
 - not too different from to the recommended allocation
 - but once again all items usually considered to have the reverse orientation are separated from all the other items

Item-Scale Allocation Summary

- varimax-based item-scale allocations for DQOLY were quite consistent with the recommended allocation
 - the recommended DQOLY scales seem appropriate to use with these subjects, but they were developed specifically for youth with diabetes
- varimax-based item-scale allocations for FACES were quite different from the recommended allocation
 - the recommended FACES scales might be inappropriate to use for families with adolescents having type 1 diabetes
- for both FACES and DQOLY, varimax rotation separated off the items with reverse orientation from the others
 - does it really identify sets of items associated with different latent constructs or just having different orientations?

Part 5

Factor Analysis Model Evaluation

- scoring factor analysis models
- choosing the # of factors
- evaluating alternative factor extraction methods
- CFA models for scales suggested by rotations
- comparison of scales
- assessing individual items
- item residual analyses

Scoring Factor Analysis Models

- using likelihood cross-validation (LCV)
 - measures how well a model estimated on portions of the data predicts the remaining data in subsets called folds
 - with the data randomly partitioned into k disjoint folds
 - based on likelihoods for data in folds using parameter estimates computed from data outside of the folds
 - using the multivariate normal likelihood as in ML factor extraction
 - multiply these deleted fold likelihoods together and normalize to the # of item responses to get the LCV score
 - larger scores mean models more compatible with data
 - scores within 1% of best are nearly optimal alternatives
 - computable for EFA and CFA models
 - using specialized SAS macros available on the Internet
- results from [10] are reported in what follows

Choosing the Number of Factors

- using ML factor extraction, the recommended # of factors is chosen for all 3 sets of items using LCV
 - 1 for CDI, 2 for FACES, and 3 for DQOLY
 - this holds for any # k of folds as long as it is not too small
 - so used k=10 for CDI and FACES and k=15 for DQOLY
- LCV seems to be a reasonable way to assess how many factors to extract
- also considered a variety of other approaches
 - including rules based on eigenvalues and penalized likelihood criteria
- the only other approach with somewhat acceptable results was the minimum BIC approach
 - which chose 1 for CDI, 2 for FACES, and 2 for DQOLY ¹²⁹

Alternative Numbers of Factors

- for CDI, 1 factor is a clear-cut choice
 - no other choices have LCV scores within 2% of best
- for FACES
 - 3 factors has a score within 1% of best
 - 1 factor has a score of just above 1% of best
- for DQOLY
 - 2, 4, and 5 factors have scores within 1% of best
- different choices for the # of factors can be competitive alternatives to the choice with the best score
 - a range of #'s of factors can have about the same effect
 - part of why choosing the # of factors is a difficult problem

Alternative Extraction Methods

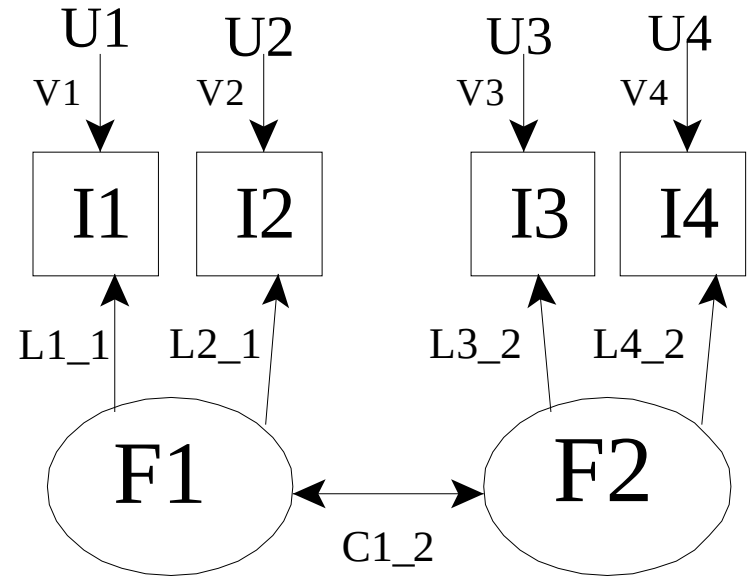
- considered a variety of factor extraction methods, factoring the correlation matrix as well as the covariance matrix when possible
 - one-step and iterated PC and PF methods
 - ML, unweighted least squares
 - image component analysis, Harris component analysis
- for all these methods, the recommended # of factors is chosen for all 3 sets of items using LCV
 - 1 for CDI, 2 for FACES, and 3 for DQOLY
 - there was also very little difference in maximum LCV scores for all of these methods
- there does not seem to be much of an impact to the choice of factor extraction procedure

Evaluation of Rotations

- rotations do not change the correlation structure of the EFA model and so cannot be directly evaluated by LCV
- but they do suggest summated scales with loadings changed to ± 1 or 0 which change the correlation structure
- so rotations can be evaluated using LCV by evaluating CFA models based on rotation-suggested scales
- considered variety of CFA models for FACES/DQOLY
 - based on rotation-suggested scales vs. on recommended scales
 - with unit (± 1) loadings vs. with estimated loadings
 - with all scales dependent vs. with all independent vs. with any subset independent and the rest dependent
- also compared these to EFA models
 - with all items allocated to all scales with estimated loadings

Example CFA Model

- this CFA model has
 - 2 factors: F1 and F2
 - 4 items: I1, I2, I3, and I4
 - items I1 and I2 load on factor F1
 - with loadings are L1_1 and L2_1
 - with unique errors U1 and U2
 - loadings L1_2 and L2_2 are 0
 - items I3 and I4 load on factor F2
 - with loadings are L3_2 and L4_2, with errors U3 and U4
 - loadings L3_1 and L4_1 are 0
 - covariance for F1, F2 is C1_2, variances of U1-U4 are V1-V4



PROC CALIS ... ;

LINEQS I1 = L1_1 F1 + U1 I2 = L2_1 F1 + U2 I3 = L3_2 F2 + U3 I4 = L4_2 F2 + U4;

COV F1 F2 = C1_2;

STD U1-U4 = V1-V4;

VAR I1-I4;

RUN;

Comparison of Scales

- treating scales as dependent was always better
 - so subsequent reported results use dependent scales
 - not so surprising since scales from the same instrument measure related latent constructs
- varimax-suggested scales with estimated loading were best overall for both FACES and DQOLY
 - other rotations were as good or at least almost as good
 - and a little better than EFA-based scales
 - so treating factors as grouped rather than as general is reasonable
- when items are reallocated 1 at a time to other scales
 - starting from varimax-suggested scale with loadings re-estimated
 - no reallocation generated an improvement for FACES
 - only one generated a very small improvement for DQOLY
 - which changes item 7's allocation to be compatible with its recommended allocation
- varimax-suggested allocations may be almost optimal¹³⁴

Comparison of Summated Scales

- recommended summated scales (with loadings of ± 1 or 0) were competitive for DQOLY but not for FACES
 - for adolescents with type 1 diabetes, the DQOLY scales seem reasonable to use but there can be a tangible penalty to using the standard FACES scales
- on the other hand, summated scales based on unrotated loadings were not competitive for both FACES and DQOLY
 - the common practice of basing scales on a rotation appears much better than basing them on unrotated loadings

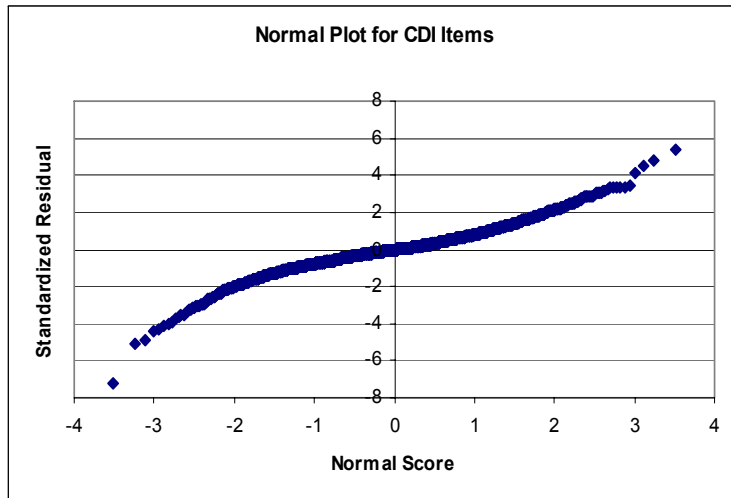
Assessing Individual Items

- to assess the value of an individual item
 - can use the % change in LCV score when an item's loadings are changed to 0 for all factors, effectively discarding it
 - the larger the % decrease, the more valuable the item
 - the larger the % increase, the more expendable the item
- for FACES and DQOLY, all items are of some value
 - with % decreases ranging from 0.04% to 3.10%
- for CDI, all but 5 items are of some value
 - 5 items 9,18,23,25,26 had very small % increases ($\leq 0.04\%$)
 - all but item 18 also have nonsignificant loadings
- there is no compelling reason to discard items from these 3 instruments
 - the removal of none provides a tangible improvement

Results for CDI

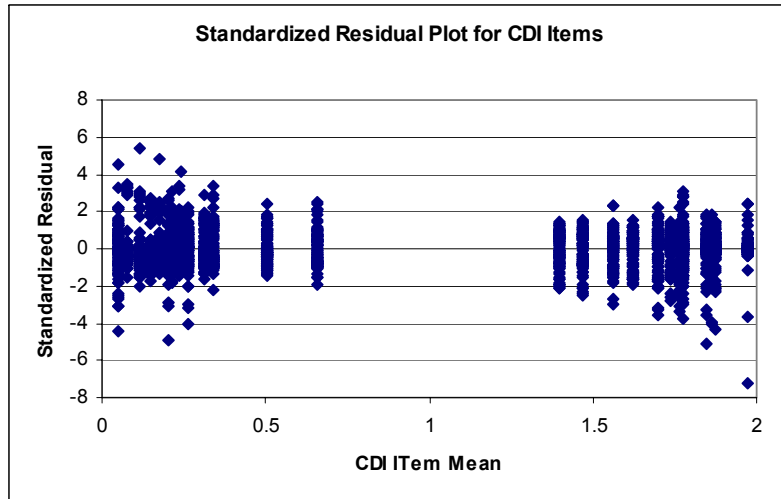
- the EFA model had the better score
 - but the recommended summated scale was competitive
- is the assumption of normality reasonable? are there any outlying item values?
 - need item residuals for this
- to assess this, standardized the item residuals to be independent and standard normally distributed
 - for the $27 \cdot 103 = 2781$ item values without reverse coding
 - evaluated the EFA model with the better LCV score
 - estimated the 27×27 covariance matrix Σ using all the data
 - to reduce the effort, computed standardized residuals for item responses from subjects in each fold separately rather than for all item responses of all subjects combined

Normal Probability Plot - CDI



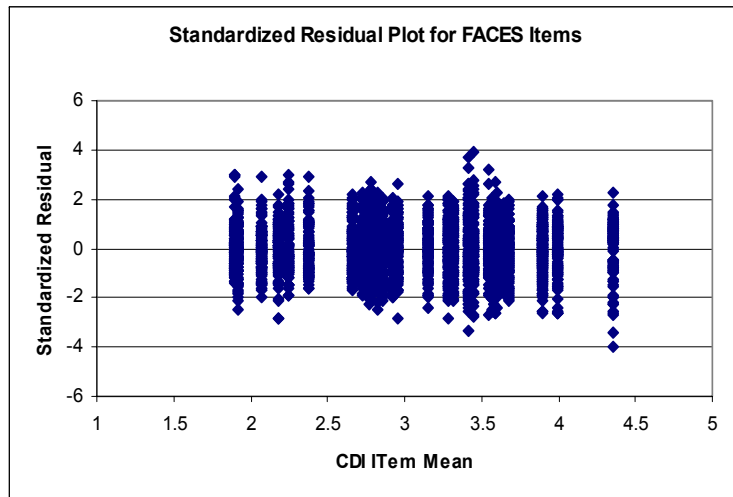
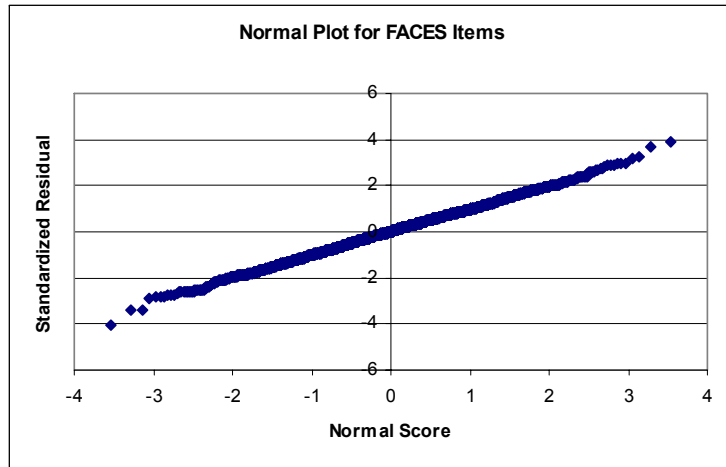
- normality assumption questionable
 - the plot is curved
- there is an extreme standardized residuals of -7.2
 - for a value of item 25 with meaning:
 - 0: nobody really loves me
 - 1: I am not sure if anybody loves me
 - 2: I am sure that somebody loves me
 - a value of 2 occurs 101 times
 - values of 0 and 1 each occur 1 time
 - the one value of 0 is the outlier
- almost all of these adolescents felt loved, so item 25 contributes little distinguishing information
 - its loading was also found not to be significantly different from zero

Standardized Residual Plot - CDI



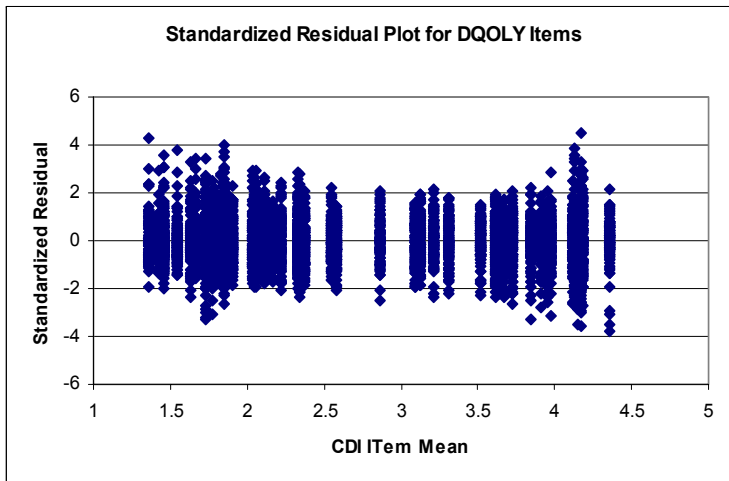
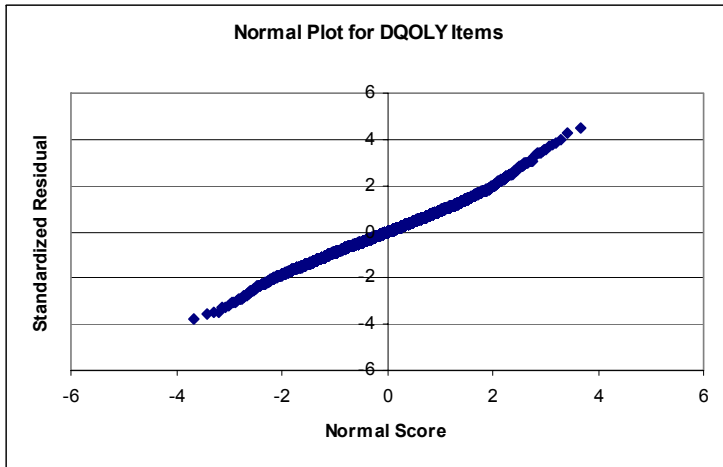
- observed CDI item means cluster near the extremes of 0 and 2 for item values
 - with residuals tending to be more outlying the closer the mean is to the extremes
- this might be why normality is questionable
- perhaps this will often hold when the range of item values is so limited

Residual Analysis - FACES



- using varimax-suggested scales with estimated loadings
 - without reverse coding items
- normality assumption appears reasonable
 - normal plot is quite straight
 - residuals quite symmetric between ± 4 and often ± 3
- observed FACES item means are all well away from the extremes of 1 and 5
- number of items responses
 - $30 \cdot 103 = 3090$

Residual Analysis - DQOLY



- using varimax-suggested scales with estimated loadings
 - without reverse coding items
- normality assumption somewhat reasonable
 - normal plot is fairly straight
 - residuals sometimes asymmetric and occasionally outside of ± 4
- observed DQOLY item means are all away from the extremes of 1 and 5
- number of items responses
 - $52 \cdot 103 = 5253$

Part 5

A Case Study in Ongoing Scale Development

Developing New Scales

- an extensive effort is required before it is possible to conduct a factor analysis
 - an initial pool of items needs to be generated
 - a primarily qualitative rather than quantitative task
 - item responses need to be collected from a large sample of subjects
 - 5-10 subjects per initial item is usually considered desirable
 - some subjects need to be interviewed at 2 points in time
 - to be able to assess test-retest reliability
- the construct validity of the new scales needs to be assessed after the factor analysis
 - do the new scales predict related quantities as expected?

Family Management Style Survey

- currently under development
 - parents of children having a chronic illness are being interviewed on how their families manage their child's chronic illness
 - interviewing is almost finished
- data currently available for 528 parents of 382 families
 - 236 families with 1 responding parent
 - 3 of these were fathers; the mothers in these families agreed to participate, but it has not yet been possible to interview them
 - 146 families with both mothers and fathers responding
- so have data for 379 mothers and 149 fathers
- complicated by the need to account for the correlation between responses of parents from the same family
 - but can analyze data from mothers/fathers separately
- only incomplete, preliminary results are currently available

The FMS Framework

- FMSS items were based on the Family Management Style (FMS) Framework
 - conceptualizes how families define and manage a child's chronic illness [17]
- 5 FMSs
 - thriving, accommodating, enduring, struggling, floundering
 - reflecting a continuum of difficulty for managing childhood chronic illness and the extent to which family members' experiences were similar or discrepant
- 3 major components of the illness experience
 - definitions of the situation, management behaviors, and perceived consequences
 - refined into 8 major themes common to all families
- FMSS items address the 3 components and 8 themes
 - so when asked to estimate the # of factors for the FMSS, the PI replied between 3 and 8

The FMSS Items

- there are 65 initial FMSS items
 - items 58-65 address issues related to the parent's spouse and so are not completed by single parents
 - will restrict the factor analysis to items 1-57 applicable to both single and partnered parents
 - for all 528 parents have 9.3 subjects per item
 - for only the 379 mothers have 6.6 subjects per item
- all items are coded from 1-5
 - 1="strongly disagree" and 5="strongly agree"
 - the interview form also included 3 other choices
 - "Not Applicable", "Don't Know", "Refused"
 - provides extra qualitative information for item assessment
 - but also increases the # of missing responses
 - only 280 of the 379 mothers provided values of 1-5 for all of items 1-57 or 4.9 subjects per item
 - very important to adjust for missing data as well as for inter-parental correlation to avoid losing so much data

Item Response Consistency

- 74 parents were retested about 2 weeks apart
 - 46 females and 28 males
 - including both parents of 24 families
- correlations between test and retest responses for each of items 1-65
 - assess the consistency of responses to items over time
 - computed for mothers separately from fathers
 - used Spearman correlations since the range 1-5 for item values was limited
 - for mothers, correlations were significantly nonzero for all items
 - for fathers, correlations were nonsignificant for 8 of the items
 - items 36($p=.057$), 22 ($p=.077$), and 7,8,18,19,29,60 ($p>.10$)
- responses for mothers were reasonably consistent across time while fathers changed responses fairly often to quite a few items (8/65 or 12.3%)

FMSS Item Means

- reported analyses that follow are for the 280 mothers who responded to all of items 1-57
- items with ceiling/floor effects are undesirable
 - means for items 1-57 ranged from 1.4 to 4.7
 - 1 item (42) had a mean < 1.5
 - most mothers strongly disagreed on 1 item
 - 5 items (23,30,39,40,52) had means > 4.5
 - most mothers strongly agreed on several items
 - for 4 items (23,30,39,52), the middle value of 3 was over 2 standard deviations from the item mean
 - these may be distinctly problematic

FMSS Item Correlations

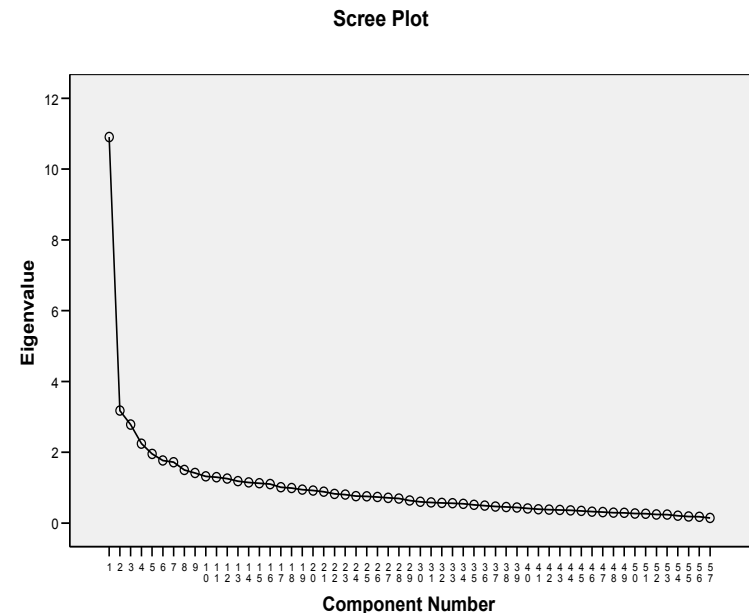
- it is usually recommended to inspect the correlation matrix for the items before factoring them [11]
 - is there a substantial # of large correlations?
 - perhaps the items are close to independent of each other and so there will be little benefit to factoring which will be deceptive, extracting factors that really do not exist
 - do they form groups?
- a daunting task when there are 57 items
 - with $57(57-1)/2=1566$ distinct correlations
- can assess if factoring provides a benefit over treating items as independent using LCV scores

Reverse Coding

- extracting 1 factor using the ML method
 - signs of the loadings suggest that 25 of the 57 items need reverse coding from the other 32 items
 - items 1,3,8,9,15,17-20,23,24,26,28,30,31,36,37,39,40,46,48,50,52,53,56
- all of these items except item 36 were considered to have been worded positively
 - item 36 was considered to have been worded neutrally
- all of the others were originally considered to have been worded negatively except for 5 items
 - items 5,6,7,45 were considered to have been worded positively
 - item 43 was considered to have been worded neutrally
 - need to check on these potential inconsistencies

of Factors

- scree plot indicates 1 to about 7 factors
- using ML factor extraction
 - BIC is minimized at 3 factors
 - LCV is maximized at 8 factors
 - but LCV scores for 3-13 factors are all within 1% of best
 - so 3 factors would be a parsimonious nearly optimal choice
- the # of factors could reasonably be one 11 different values
 - why choosing it can be difficult using conventional methods
- but results are consistent with the FMS Framework



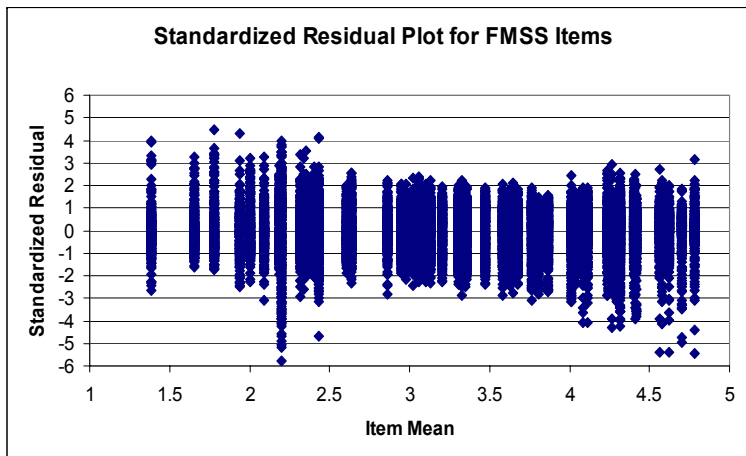
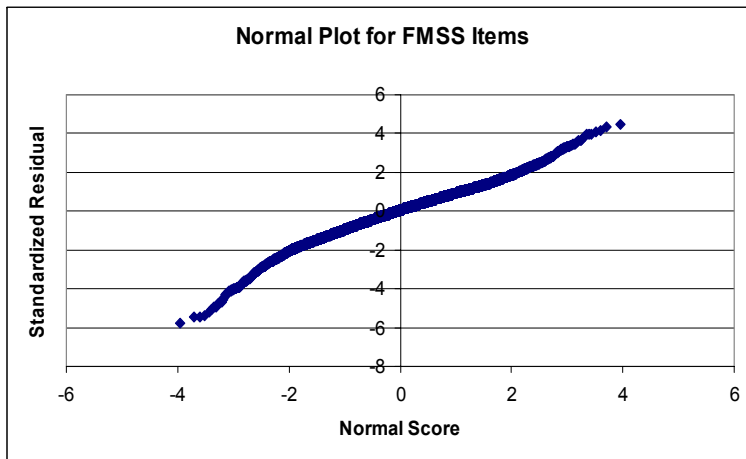
Comparison of Scales

- using the 8-factor solution with best the LCV score
- the independent errors model had 9.8% lower score
 - so there is a distinct benefit to factoring the items
- using estimated loadings
 - the EFA model had a better LCV score
 - treating factors as general with all items loading on all factors
 - than the associated CFA model for varimax-suggested scales with estimated loadings
 - treating factors as grouped with items loading only on one of the factors
 - but the scores were not too different
 - with only a 0.3% decrease in LCV
 - so treating the factors as group factors seems reasonable

Comparison of Scales

- using unit loadings (i.e. summated scales)
 - the LCV score decreased by a little over 2%
 - can be a tangible penalty to using summated scales
- using the model suggested by the FMS Framework
 - with the 57 items allocated to 7 theory-based factors
 - items 59-65 correspond to a 8th theory-based factor, but these were not used in the analysis
 - the model with estimated loadings was competitive
 - LCV score about 1.5% lower than the best overall score
 - so basing scales on theory may be reasonable ¹⁵³

Residual Analysis - FMSS



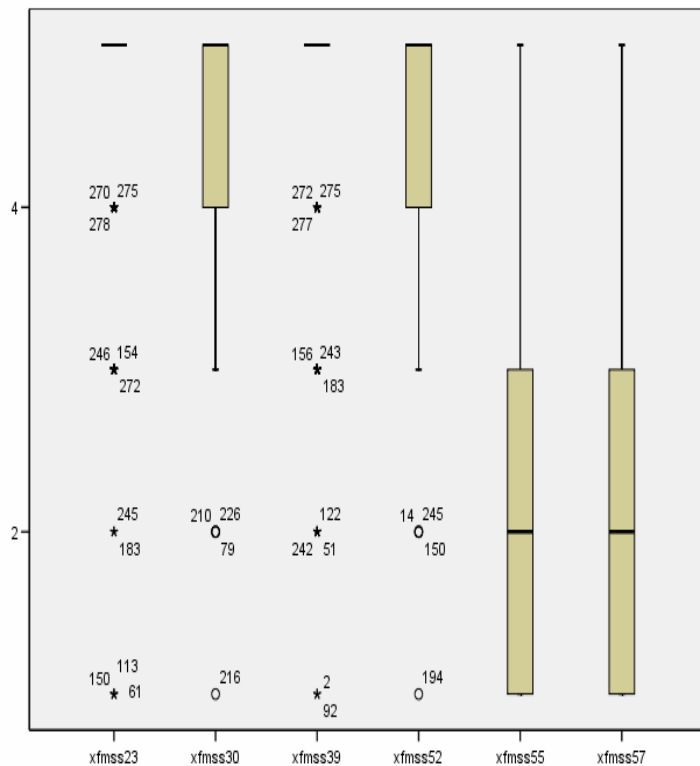
- using varimax-suggested scales with estimated loadings for 8 factors
- normality assumption somewhat questionable
 - normal plot curved at low end
 - residuals can be skewed, but more to the low end than the high end
 - lower negative values are due to larger means, i.e., a tendency to respond as strongly disagree more often
- extreme residuals for 6 items
 - with absolute value over 4.5
 - items 23,30,39,52 as identified before as well as items 55,57
- number of items responses
 - $57 \cdot 280 = 15960$

Item Removal

- removing either item 55 or item 57 imposes a tangible penalty in reduced LCV score
 - removing either generates a 1.1% decrease in LCV
 - while they may generate large residuals, they still have value
- removing items 23,30,39,52 does not impose a tangible penalty
 - removing them one at a time generates decreases in LCV of less than 0.5%
 - removing all of them together generates a decrease in LCV of 0.6%
 - these items all seem expendable
- still need to assess the impact of removal of the other items

Item Boxplots

- items 23,30,39,52 are highly skewed at the low end



- primarily strongly disagree with responses close to strongly agree outlying
- items 55,57 are highly skewed the other way
 - primarily strongly agree with responses close to strongly disagree quite a bit less likely, but not outlying

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