

Converting English to Boolean

1. Identify phrases

The air conditioner should be turned on
if and only if the temperature is greater
than 75°, the time is between 8a.m.
and 5 p.m., and it is not a holiday.

F = air conditioner should be turned on

A = temperature is greater than 75°

B = time is between 8a.m. and 5 p.m

C = it is a holiday

3. Construct a Boolean Expression

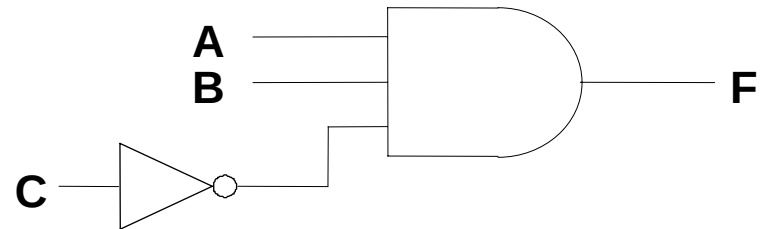
$$F = A \cdot B \cdot C'$$

2. Identify connective words

The air conditioner should be
turned on if and only if the
temperature is greater than 75°,
the time is between 8a.m. and 5
p.m., and it is not a holiday.

= implied
and

4. Draw the Network



Converting English to Boolean

- Be careful: Boolean algebra is precise, English is not.

The roads will be very slippery if it snows^A
or rains^B and there is oil^C on the road.

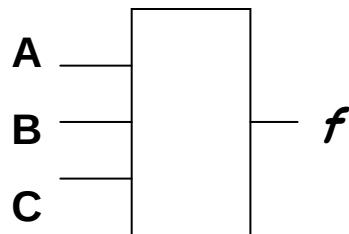
$$F = A + BC$$

OR

$$F = (A + B) C$$

Design a Logic Network from a Truth Table

A	B	C	f	f'
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0



1. Write the equation term by term

$$f = A'B'C + AB'C + ABC' + ABC$$

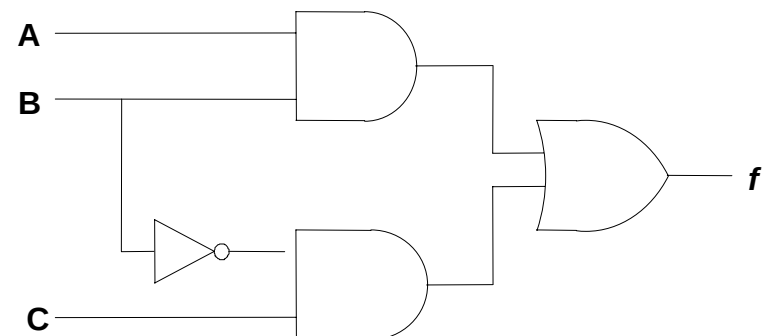
2. Simplify the equation

$$f = A'B'C + AB'C + ABC' + ABC$$

$$f = (A + A') B'C + AB (C + C')$$

$$f = AB + B'C$$

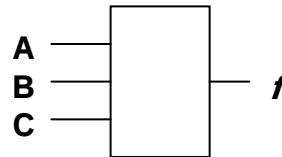
3. Draw the logic network



Design a Logic Network from a Truth Table

Sometimes it is easier to work with the product of sums

A	B	C	f	f'
0	0	0	1	0
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0



$$f = A'B'C' + A'B'C + A'BC + AB'C' + AB'C + ABC$$

$$f' = A'BC' + ABC'$$

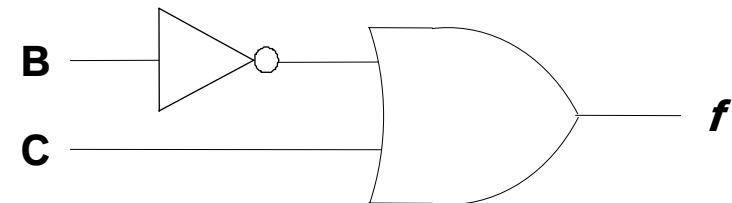
$$f = (A + B' + C) (A' + B' + C)$$

2. Simplify the equation

$$f = (A + B' + C) (A' + B' + C)$$

$$f = B' + C$$

3. Draw the logic network



Minterm Expansion

- Definition: a Minterm of n variables is a product of n literals where each variable appears exactly once in true or complemented form.

- Are these minterms of 4 variables?

$abcd$

Yes

$ab'c$

No, only 3 literals

$ab'cd'$

Yes

$a'bc'bd$

No, b more than once

Minterms have names

A	B	C	Minterms
0	0	0	$A'B'C' = m_0$
0	0	1	$A'B'C = m_1$
0	1	0	$A'BC' = m_2$
0	1	1	$A'BC = m_3$
1	0	0	$AB'C' = m_4$
1	0	1	$AB'C = m_5$
1	1	0	$ABC' = m_6$
1	1	1	$ABC = m_7$

Note: the order of the variables is significant.

Minterm Expansion

Any function can be written as a sum of minterms. This is called a:

Standard Sum of Products

A	B	C	f	Minterms
0	0	0	0	m_0
0	0	1	1	m_1
0	1	0	0	m_2
0	1	1	0	m_3
1	0	0	0	m_4
1	0	1	1	m_5
1	1	0	1	m_6
1	1	1	1	m_7

$$f(A,B,C) = m_1 + m_5 + m_6 + m_7$$

$$f(A,B,C) = m_1 + m_5 + m_6 + m_7$$

Can also be written:

$$f(A,B,C) = \sum m(1, 5, 6, 7)$$

- There is only one way to write a Minterm Expansion
- No simplification or manipulation allowed

Maxterm Expansion

- Definition: a Maxterm of n variables is a sum of n literals where each variable appears exactly once in true or complemented form.

- Are these maxterms of 4 variables?

$(a+b+c+d)$

Yes

$(a+b'+c+d')$

Yes

$(a+b'+d')$

No, only 3 literals

$(a'+b+c'+a'+d)$

No, a' more than once

Maxterms have names too

A	B	C	Minterms	Maxterms
0	0	0	$A'B'C' = m_0$	$A + B + C = M_0$
0	0	1	$A'B'C = m_1$	$A + B + C' = M_1$
0	1	0	$A'BC' = m_2$	$A + B' + C = M_2$
0	1	1	$A'BC = m_3$	$A + B' + C' = M_3$
1	0	0	$AB'C' = m_4$	$A' + B + C = M_4$
1	0	1	$AB'C = m_5$	$A' + B + C' = M_5$
1	1	0	$ABC' = m_6$	$A' + B' + C = M_6$
1	1	1	$ABC = m_7$	$A' + B' + C' = M_7$

Note: the order of the variables is significant.

Maxterm Expansion

Any function can be written as a product of maxterms. This is called a:

Standard Product of Sums

A	B	C	f	Maxterms
0	0	0	0	M_0
0	0	1	1	M_1
0	1	0	0	M_2
0	1	1	0	M_3
1	0	0	0	M_4
1	0	1	1	M_5
1	1	0	1	M_6
1	1	1	1	M_7

Note the terms for $f = 0$ must be included in the product

$$f(A,B,C) = M_0 M_2 M_3 M_4$$

$$f(A,B,C) = M_0 M_2 M_3 M_4$$

Can also be written:

$$f(A,B,C) = \prod M(0, 2, 3, 4)$$

- There is only one way to write a Maxterm Expansion
- No simplification or manipulation

Minterm / Maxterm Relationships

For any function, if a minterm is in the Minterm Expansion, the corresponding maxterm is not in the Maxterm Expansion.

A	B	C	f	f'
0	0	0	0	1
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

$$f(A,B,C) = m_1 + m_2 + m_5 + m_6 + m_7$$

$$f(A,B,C) = M_0 M_3 M_4$$

For any function (f), if a minterm is in the Minterm Expansion, the same minterm is not in the Minterm Expansion for (f').

$$f(A,B,C) = m_1 + m_2 + m_5 + m_6 + m_7$$

$$f'(A,B,C) = m_0 + m_3 + m_4$$

For any function (f), if a maxterm is in the Maxterm Expansion, the same maxterm is not in the Maxterm Expansion for (f').

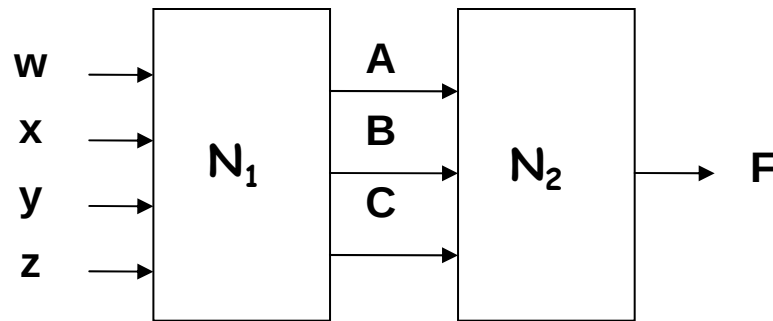
$$f(A,B,C) = M_0 M_3 M_4$$

$$f'(A,B,C) = M_1 M_2 M_5 M_6 M_7$$

Canonical and Standard Forms

- A function is said to be in Canonical Form if it is expressed either in Sum of Minterms form or Product of Maxterms form.
- A Standard form can be obtained upon simplifying a Sum of Minterms or Product of Maxterms expression. A term in a SOP (Sum Of Products) or POS (Product of Sums) form need not contain all the variables of a function.

Incompletely Specified Functions



Known: N_1 does not generate outputs A B C with values of 101 or 111

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	X
1	1	0	1
1	1	1	X

A	B	C	F	F_1	F_2	F_3	F_4
0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	1
0	1	0	0	0	0	0	0
0	1	1	0	0	0	0	0
1	0	0	1	1	1	1	1
1	0	1	X	0	0	1	1
1	1	0	1	1	1	1	1
1	1	1	X	0	1	0	1

$$F_1 = A'B'C + AB'C' + ABC'$$

$$F_1 = A'B'C + AC'$$

$$F_2 = A'B'C + AB'C' + ABC' + ABC$$

$$F_2 = A'B'C + AC'(B' + B) + AB(C' + C)$$

$$F_2 = A'B'C + AC' + AB \quad \text{Worst than } F_1$$

$$F_3 = A'B'C + AB'C' + AB'C + ABC'$$

$$F_3 = A'C(B + B') + AB'(C' + C)$$

$$F_3 = A'C + AB' \quad \text{Better than } F_1$$

$$F_4 = A'B'C + AB'C' + AB'C + ABC' + ABC$$

$$F_4 = (A' + A)B'C + AB'(C' + C) + AB(C' + C)$$

$$F_4 = B'C + AB' + AB = B'C + A \quad \text{Best}$$

Minterm and Maxterm Representation

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	X
1	1	0	1
1	1	1	X

- Use $d_1 d_6 \dots$ to represent don't care minterms

$$F = m_1 + m_4 + m_6 + d_5 + d_7$$

$$F = \sum m(1, 4, 6) + \sum d(5, 7)$$

- Use $D_1 D_6 \dots$ to represent don't care maxterms

$$F = M_0 M_2 M_3 D_5 D_7$$

$$F = \prod M(0, 2, 3) \prod D(5, 7)$$