

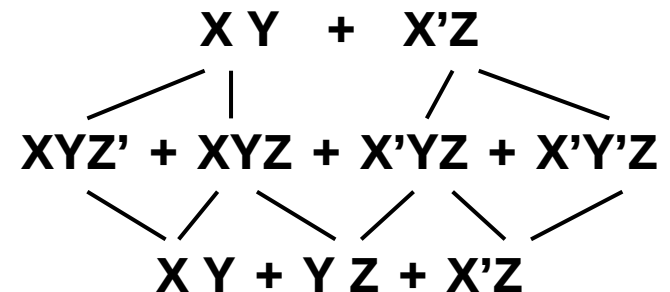
Consensus Theorem

$$X Y + X' Z + Y Z = X Y + X' Z$$

What is really going on?

The **YZ** term is redundant

This Theorem is very useful for eliminating extra terms



Proof:

$$\begin{aligned} X Y + X' Z + Y Z &= X Y + X' Z + (X + X') Y Z = (X Y + X Y Z) + (X' Z + X' Y Z) \\ &= X Y (1 + Z) + X' Z (1 + Y) = X Y + X' Z \end{aligned}$$

Consensus Theorem – Dual

$$(X + Y)(X' + Z)(Y + Z) = (X + Y)(X' + Z)$$

Proof: If an expression is true, its dual is true

Examples

Ex.: Simplify: $BC'D' + ABC' + AC'D$

Look for X and X'

$$BC'D' + ABC' + AC'D$$

$$BC'D' + ABC' + AC'D$$

$$BC'D' + ABC'C' + AC'D$$

$$BC'D' + AC'D$$

Ex.: Simplify: $A'B'C + AD' + BD' + CD'$

Factor

$$A'B'C + (A + B)D' + CD'$$

Notice $(A'B')' = (A + B)$

$$A'B'C + (A + B)D' + CD'$$

$$\begin{matrix} X & Y & & X' & Z & Y & Z \end{matrix}$$

$$A'B'C + (A + B)D'$$

$$A'B'C + AD' + BD'$$

Methods of Algebraic Simplification

- Combine terms

$$\begin{array}{c} ABC' + A'B'C + A'BC' \\ BC' + A'B'C \end{array}$$

$$\text{Use } XY + XY' = X$$

- Eliminate terms

$$\begin{array}{c} A'B + BC + A'BD \\ A'B + BC \end{array}$$

$$\text{Use } X + XY = X$$

- Eliminate literals

$$\begin{array}{l} A'B + A'B'C'D' + ABCD' \\ A'(B + B'C'D') + ABCD' \\ A'(B + C'D') + ABCD' \\ A'B + A'C'D' + ABCD' \\ A'C'D' + B(A' + ACD') \\ A'C'D' + B(A' + CD') \\ A'B + BCD' + A'C'D' \end{array}$$

$$\text{Use } X + X'Y = X + Y$$

- Add redundant terms

Add Redundant Terms Example

Simplify:

$$ab + a'bc' + bc$$

What terms can be removed?

Look for X and X'

$$ab + a'bc' + bc$$

Generate a consensus term

$$ab + a'bc' + bc + bbc'$$

Use $XX = X$ and $XY + XY' = X$

$$ab + a'bc' + b$$

Use $X + XY = X$

$$b$$

A Comprehensive Example

Simplify:

$$A'C'D' + AC' + BCD + A'CD' + A'BC + AB'C'$$

Use $XY + XY' = X$

$$A'C'D' + AC' + BCD + A'CD' + A'BC + AB'C'$$

$$A'D' + AC' + BCD + A'BC + AB'C'$$

Use $X + XY = X$

$$A'D' + AC' + BCD + A'BC + AB'C'$$

$$A'D' + AC' + BCD + A'BC$$

$$A'D' + AC' + BCD + A'BC$$

$$A'D' + AC' + BCD$$

Proving an Equation

- Construct a truth table for both sides
 - Tedious, not very elegant
 - Computers love this method
- Manipulate one side algebraically to equal the other
- Manipulate both sides algebraically to the same expression
- Perform the same operation on both sides
 - Valid Operations
 - Complement or Invert
 - Construct the Dual of both sides
 - Invalid Operations
 - Add a term to both sides (not reversible)
 - Subtract a term from both sides (not true)
 - Multiply (AND) both sides by a term (not reversible)
 - Eliminate a multiplier from both sides (not true)

Proving an Equation

■ Add a term to both sides

Prove $y = z$

$$\begin{array}{l} \text{Add 1 to both sides} \\ y + 1 = z + 1 \\ \text{Use } X + 1 = 1 \\ 1 = 1 \end{array}$$

Therefore $y = z$

But not really, you can not add an arbitrary term to both sides.

This does not prevent you from adding terms:

$$y + y = z + z$$

■ Multiply (AND) both sides by a term

Prove $y = z$

$$\begin{array}{l} \text{Multiply both sides by 0} \\ y \cdot 0 = z \cdot 0 \\ \text{Use } X \cdot 0 = 0 \\ 0 = 0 \end{array}$$

Therefore $y = z$

But not really, you can not multiply both sides by an arbitrary term

This does not prevent you from multiplying terms:

$$y \cdot y = z \cdot (z + z')$$

■ Subtracting a term from both sides

$$x + y = x + z$$

Subtract x from both sides

$$\begin{array}{l} y = z \\ \text{Suppose } x = 1, y = 0 \text{ and } z = 1 \\ x + y = x + z = 1 \\ y = z \\ 0 = 1 \end{array}$$

Boolean algebra does not allow subtraction from both sides of an equation.

■ Eliminate a multiplier from both sides

$$x \cdot y = x \cdot z$$

Eliminate x from both sides

$$\begin{array}{l} y = z \\ \text{Suppose } x = 0, y = 0 \text{ and } z = 1 \\ x \cdot y = x \cdot z = 0 \\ y = z \\ 0 = 1 \end{array}$$

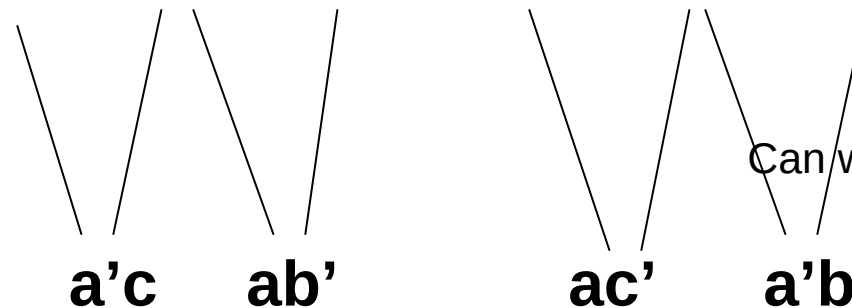
Boolean algebra does not allow multipliers to be deleted from both sides of an equation.

Proving an Equation is False

- Show one combination of values which is false
- Use Boolean algebra manipulation
 - Reduce to sum of products form
 - Compare the 2 sides
 - Add missing terms to one side (consensus theorem)
 - Remove terms from one side (consensus theorem)

Prove this Equation is True

$$a'b + b'c + c'a = ab' + bc' + ca'$$



What terms are the same?

Can we add terms - Look for X and X'

$$ab' + a'b + a'c + b'c + ac' = ab' + ac' + a'b + bc' + a'c$$

X Y'

X' Z Y Z

X Y

X' Z

Z Y

What terms are different?

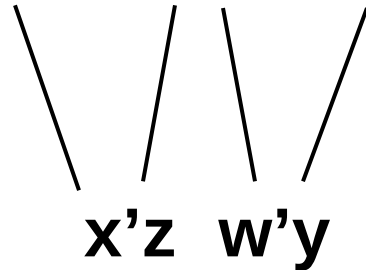
Can they be eliminated?

$$ab' + a'b + a'c + ac' = ab' + ac' + a'b + a'c$$

It is true

A Final Example

$$w'x' + x'y' + yz + w'z'$$



What terms can be removed?

What terms can be generated?

Look at groupings

$$\underline{w'x' \quad x'z \quad w'z'} \quad \underline{w'y \quad x'y' \quad w'x'} \quad \underline{yz}$$



$$x'y' + yz + w'z' + x'z + w'y$$

Remove $w'x'$



Then remove the generated term

$$x'y' + yz + w'z'$$