

Boolean Expressions

Boolean expressions are made up of variables and constants combined by AND, OR and NOT

Examples: $[A B (C + B') + D]$ $B \cdot E$ C' 1

Literals each instance of a variable or constant

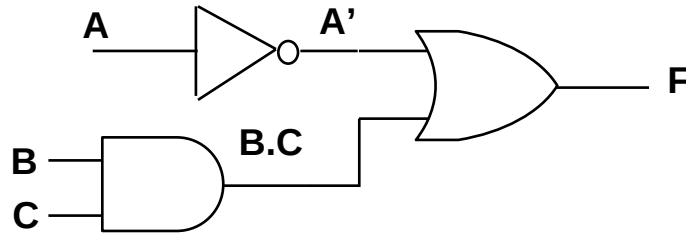
This expression has 4 variables and 10 literals:

$$a'bd + bcd + ac' + a'd'$$

Boolean Expressions

Each Boolean expression has a corresponding realization with logic gates.

$$F = A' + B C$$



Each Boolean expression can be specified by a truth table which lists all possible combinations of the values of all variables in the expression.

A	B	C	A'	B C	F = A' + B C
0	0	0	1	0	1
0	0	1	1	0	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	0	0	0
1	1	0	0	0	0
1	1	1	0	1	1

Proof Using Truth Tables

Truth Tables can be used to prove that 2 Boolean expressions are equal.

If the 2 expressions have the same value for all possible combinations of variables, they are equal.

$$X Y' + Y = X + Y$$

X	Y	Y'	X Y'	X Y' + Y	X + Y
0	0	1	0	0	0
0	1	0	0	1	1
1	0	1	1	1	1
1	1	0	0	1	1

↖ = ↗

These two expressions are equal.

Basic Boolean Algebra Theorems

Here are the first 5 of Boolean Algebra theorems and postulates that we will study and use.

$X + 0 = X$	$X \cdot 1 = X$	Postulates
$X + X' = 1$	$X \cdot X' = 0$	
$X + 1 = 1$	$X \cdot 0 = 0$	Theorems
$X + X = X$	$X \cdot X = X$	
$(X')' = X$		

Each can easily be proved by showing it valid for $X = 0$ and $X = 1$. Also theorems may be proved by using postulates.

Basic Boolean Algebra Theorems

While these laws don't seem very exciting, they can be very useful in simplifying Boolean expressions:

Simplify: $(M N' + M' N) P + P' + 1$

$X + 1 = 1$ and X can be anything.

In this case, $X = (M N' + M' N) P + P'$

and the expression = 1

Basic Boolean Algebra Theorems

- Commutative Laws

$$X \cdot Y = Y \cdot X$$

$$X + Y = Y + X$$

- Associative Laws

$$(X \cdot Y) \cdot Z = X \cdot (Y \cdot Z) = X \cdot Y \cdot Z$$

$$(X + Y) + Z = X + (Y + Z) = X + Y + Z$$

- Distributive Law

$$X(Y + Z) = XY + XZ$$

- Just like regular algebra

Proof of Distributive Law

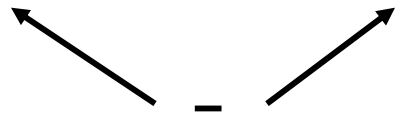
$$X (Y + Z) = X Y + X Z$$

Prove with a truth table:

X	Y	Z	Y + Z	X (Y + Z)	X Y	X Z	X Y + X Z
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

Just like regular algebra

=



Other Distributive Law

$$X + YZ = (X + Y)(X + Z)$$

wow!

Proof:

$$\begin{aligned}(X + Y)(X + Z) &= X(X + Z) + Y(X + Z) && 2-1 \\ &= XX + XZ + YX + YZ && 2-2 \\ &= X + XZ + XY + YZ && 2-3 \\ &= X \cdot 1 + XZ + XY + YZ && 2-4 \\ &= X(1 + Z + Y) + YZ && 2-5 \\ &= X \cdot 1 + YZ && 2-6 \\ &= X + YZ && 2-7\end{aligned}$$

NOT like regular algebra!

Venn Diagrams

- Venn diagrams may be used to prove Boolean algebra theorems and logic expressions.

2 Variable Venn Diagrams

