Introduction to Complements

- **Complements are used to simplify the subtraction operation and for logical manipulations.**
- **For each base (r), there are two complements:**
  - **(r-1)'s complement, also called diminished radix complement**
  - **r's complement, also called radix complement.**
- **(r-1)'s complement: For any integer number N in base r with number of digits equal n we define:**

  (r-1)'s complement of N = $(r^n - 1) - N$

- **r's complement of N: For any integer number N in base r with number of digits equal n we define:**

  r's complement of N = $r^n - N$ = (r-1)'s complement +1

Diminished Radix Complement

- **Decimal number complements:**
  - **9's complement of the decimal number N = $(10^n - 1) - N$**
    $= n\ (9\text{'s}) - N$

    i.e. {subtract each digit from 9}

  - **Example 9's complement of 45879 is 54120**

- **Binary number complement:**
  - **Similar to the decimal number system, 1's complement of the binary number N = $(2^n - 1) - N = n\ (1\text{'s}) - N$**

  - **Example 1's complement of 100100101 is 011011010 which can be obtained by replacing each one by a zero and each zero by one**

Radix Complement

- **10's complement of the decimal number N = 10^n – N = 9's complement + 1**
  - **Example 10's complement of 45879 is 54121**
  - **Example 10's complement of 452300 is 547700**
- **Rule: To find the 10's complement of a decimal number leave all leading zeros unchanged. Then subtract the first non-zero digit from 10 and all the remaining digits from 9's**
- **The 2's complement of a binary number is defined in a similar way.**
  - **Example: Find the 1's and 2's complements of 100010100**
  - **Answer 1's complement is 011101011**
  - **2's complement is 011101100**
- **Rule: To find the 2's complement of a binary number leave all leading zeros and first one unchanged. Then invert the remaining digits**

Subtraction Using r's Complement

- **To find M-N in base r, we add M + r's complement of N**
  - **Result is M + (r^n – N)**
  - **i) If M > N then result is M – N + r^n (r^n is an end carry and can be neglected.**
  - **ii) If M < N then result is r^n –(N-M) which is the r's complement of the result**

- **Example: Subtract (76425 – 28321) using 10's complements**

  Answer:

$$
\begin{array}{rr}
 & 7\,6\,4\,2\,5 \\
+ & \underline{7\,1\,6\,7\,9} \\
1 & 4\,8\,1\,0\,4
\end{array}
\quad \text{(10's comp. of 28321)}
$$

Discard

Therefore the difference is 48104 after discarding the end carry

Examples

- **Example: subtract (28531 – 345920) using 10's complement**

  Answer: It is obvious that the difference is negative. We also have to work with the same number of digits, when dealing with complements

```
   0 2 8 5 3 1
 + 6 5 4 0 8 0   (10's complement of 345920)
   6 8 2 6 1 1
```

  Since we have no end carry the difference is negative and is equal to the 10's complement of the answer.

  Difference is - 317389

- **Example: subtract the following binary numbers (11010011 – 10001100) using 2's complement**

  Answer :

```
                1 1 0 1 0 0 1 1
Discard       + 0 1 1 1 0 1 0 0   (2's complement of 10001100)
              1 0 1 0 0 0 1 1 1
```

  The difference is positive and is equal to 01000111

Subtraction Using the (r-1)'s Complements

- **The same rules used with the subtraction using the r's complement apply to subtraction using the (r-1)'s complements.**
- **The only difference is that when an end carry is generated, it is not discarded but added to the least significant digit of the result.**
- **Also, if no end carry is generated, then the answer is negative and is in the (r-1)'s complement form.**

- **Example: subtract (11010011 – 10001100) using 1's complement.**

Answer :

$$
\begin{array}{r l}
1\,1\,0\,1\,0\,0\,1\,1 & \\
+\ \underline{0\,1\,1\,1\,0\,0\,1\,1} & \text{(1's complement of 10001100)} \\
1\ 0\,1\,0\,0\,0\,1\,1\,0 & \\
1 & \\
\hline
0\,1\,0\,0\,0\,1\,1\,1 &
\end{array}
$$

The difference is positive and is equal to 01000111