

Numbers in Various Bases

Any number N can be represented in any base (r). As:

$$\dots a_3 a_2 a_1 a_0 . a_{-1} a_{-2} \dots$$

$$\therefore N = \dots + a_3 * r^3 + a_2 * r^2 + a_1 * r^1 + a_0 * r^0 + a_{-1} * r^{-1} + a_{-2} * r^{-2} + \dots$$

Where “r” is the base (or the radix) of the number system.

Examples:

$$527.46_{10} = 5 \times 10^2 + 2 \times 10^1 + 7 \times 10^0 + 4 \times 10^{-1} + 6 \times 10^{-2}$$

$$527.46_8 = 5 \times 8^2 + 2 \times 8^1 + 7 \times 8^0 + 4 \times 8^{-1} + 6 \times 8^{-2}$$

$$527.46_7 = \text{illegal} - \text{why?}$$

Binary Number Representation

- In a digital system, values take on only two states
 - High/low voltage
 - Horizontal/vertical polarization
 - Up/down spin
 - The choice depends on the technology chosen
 - For a number of years high/low voltage has been the most common
- This leads to the use of binary number systems since a base-2 number system has only two digits: {0, 1}
- In digital systems, numbers are represented in binary (base-2)

Binary Numbers

$$101011.11_2 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ + 1 \times 2^{-1} + 1 \times 2^{-2}$$

Convert to Base 10:

$$= 32 + 0 + 8 + 0 + 2 + 1 + \frac{1}{2} + \frac{1}{4}$$

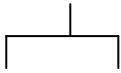
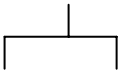
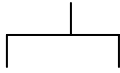
$$= 43.75_{10}$$

Octal Numbers

Octal Numbers are numbers to the base 8

$$472_8 = 4 \times 8^2 + 7 \times 8^1 + 2 \times 8^0$$

Convert from Octal to Binary and back

			
4	7	2	octal
100	111	010	binary

Hexadecimal Numbers

Hexadecimal Numbers are numbers to the base 16

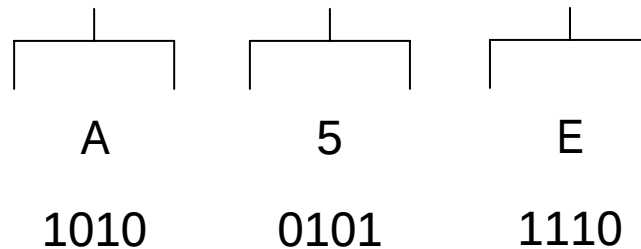
We need additional symbols to represent digits greater than 9

Value	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Symbol	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

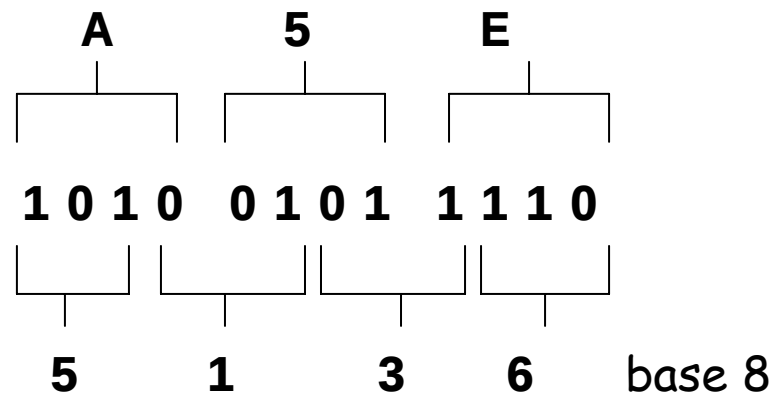
$$\mathbf{A5E_{16} = 10 \times 16^2 + 5 \times 16^1 + 14 \times 16^0 = 2560 + 80 + 14 = 2654_{10}}$$

Hexadecimal Numbers

Conversion from Hexadecimal to Binary



Conversion from Hexadecimal to Octal is similar



Numbers with Different Bases

Hex.		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
Decimal		00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
Octal		00	01	02	03	04	05	06	07	10	11	12	13	14	15	16	17
Binary		0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111

What you observe when we move from higher radix to lower radix?

Base r Conversion To/From Decimal

Conversion From Base r To Decimal:

Use the positional notation used in the previous examples to convert from any base to decimal.

$$N_{10} = \dots + a_3 * r^3 + a_2 * r^2 + a_1 * r^1 + a_0 * r^0 + a_{-1} * r^{-1} + a_{-2} * r^{-2} + \dots$$

Conversion From Decimal To Base r :

The number is separated into integer part and a fraction. The integer is successively divided by the new base, keeping track of the remainder, until the quotient is zero. The fraction is multiplied by the base, keeping track of the generated integer part, until the required accuracy is reached.

Examples

$$1) 472.25_8 = 4 \times 8^2 + 7 \times 8^1 + 2 \times 8^0 + 2 \times 8^{-1} + 5 \times 8^{-2} = 314.328125_{10}$$

$$2) 101011.11_2 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} = 43.75_{10}$$

$$3) 23.25_{10} = 10111.01_2$$

a) Integer Part

	0	1	2	5	11	23(/2)
Remainder 	1	0	1	1	1	

$$\therefore 23_{10} = 10111_2$$

b) Fraction Part

0.25	0.5	.0
.	0	1

 Integer part

$$\therefore 0.25_{10} = 0.01_2$$

Convert from Decimal to Binary (Alternate Method)

Convert 23.25_{10}
to Binary

Subtract the
powers of 2:

$2^5 = 32$	23.25 <u>- 0</u> 23.25	0
$2^4 = 16$	<u>- 16</u> 7.25	1
$2^3 = 8$	<u>- 0</u> 7.25	0
$2^2 = 4$	<u>- 4</u> 3.25	1
$2^1 = 2$	<u>- 2</u> 1.25	1
$2^0 = 1$	<u>- 1</u> 0.25	1
$2^{-1} = 0.5$	<u>- 0</u> 0.25	0
$2^{-2} = 0.25$	<u>- 0.25</u> 0	1
		
		10111.01_2