



الفريق العربي للبرمجة

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Solution to a generating function problem

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I. The Problem

A ship carries 48 flags, 12 each of the colour red, white, blue, and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships.

- 1) How many of these signals use an even numbers of blue flags and an odd number of black flags?
- 2) How many of the signals have at least three white flags and or no white flags at all?

II. Solution to problem 1

In generating function terms we present the equation as follow :

$$f(x) = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots\right)^2 \left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots\right) \left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots\right) \quad (1)$$

We know that

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

thus

$$f(x) = (e^x)^2 \left(\frac{e^x + e^{-x}}{2}\right) \left(\frac{e^x - e^{-x}}{2}\right) = \left(\frac{1}{4}\right) (e^{2x} - e^{-2x}) = \frac{1}{4} (e^{4x} - 1) = \frac{1}{4} \left(\sum_{i=0}^{\infty} \frac{(4x)^i}{i!} - 1\right) = \left(\frac{1}{4}\right) \sum_{i=1}^{\infty} \frac{(4x)^i}{i!} \quad (2)$$

thus the solution is:

$$\frac{1}{4}(4^{12}) = 4^{11} = 4194304$$

III. Solution to problem 2

comming soon

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