

Solution of Quiz 01

Consider the partition $P = \{0, 1/2, 1\}$ and $f(x) = \frac{1}{x^2 + 1}$.

Since f is decreasing, then we have

$$\sigma^-(f; P) = \frac{1}{2}f\left(\frac{1}{2}\right) + \frac{1}{2}f(1) = \frac{1}{2}\left(\frac{4}{5}\right) + \frac{1}{2}\left(\frac{1}{2}\right) = \frac{13}{20} ;$$

$$\sigma^+(f; P) = \frac{1}{2}f(0) + \frac{1}{2}f\left(\frac{1}{2}\right) = \frac{1}{2} + \frac{1}{2}\left(\frac{4}{5}\right) = \frac{9}{10}$$

we get the desired inequalities.
